Endogenous Antitrust Enforcement
in the Presence of a Corporate Leniency Program[*]

Joseph E. Harrington, Jr.
Dept. of Bus. Econ. & Public Policy
The Wharton School
University of Pennsylvania
Philadelphia, PA 19102
harrij@wharton.upenn.edu

Myong-Hun Chang
Department of Economics
Cleveland State University
Cleveland, OH 44115
m.chang@csuohio.edu

30 November 2012

Abstract

Constructing a birth and death model of cartels, this paper examines the impact of a corporate leniency program on the frequency of cartels in a population of industries. An innovative feature of the model is taking account of how a leniency program impacts enforcement through non-leniency means, specifically, the effectiveness of a competition authority in prosecuting cases without a leniency applicant. It is shown that a leniency program is assured of lowering the cartel rate when leniency cases take up sufficiently few competition authority resources or when enforcement was initially very weak. When leniency cases are just as intensive to prosecute and penalties are sufficiently low then a leniency program is not only ineffective but actually *raises* the cartel rate because of its deleterious effect on non-leniency enforcement. It is also found that the effect of a leniency program can vary significantly across industries. Finally, measuring the performance of a leniency program using the number of leniency applications is shown to be problematic because a leniency program can lower the cartel rate while generating no applications and raise the cartel rate while generating many applications.

1

1 Introduction

The 1993 revision of the Corporate Leniency Program of the U.S. Department of Justice's Antitrust Division gives a member of a cartel the opportunity to avoid government penalties if it is the first to fully cooperate and provide evidence. In the U.S., this program is arguably the most significant policy development in the fight against cartels since the Clayton Act instituted private treble damages in 1914. As reported by Antitrust Division officials, the leniency program is the primary generator of cartel cases, and the information provided by those admitted to the program has been instrumental in securing the convictions of other cartel members. Deputy Assistant Attorney General Scott Hammond stated in 2005:[1]

> The Antitrust Division's Corporate Leniency Program has been the Division's most effective investigative tool. Cooperation from leniency applicants has cracked more cartels than all other tools at our disposal combined.

The widespread usage of the leniency program in the U.S. soon led to the adoption of similar programs in other countries. In 1996, the European Commission (EC) instituted a leniency program and a decade later 24 out of 27 EU members had one. Globally, leniency programs are now present in more than 50 countries and jurisdictions.[2]

In light of the widespread adoption and usage of leniency programs, a considerable body of scholarly work has developed to understand these programs and assess how they can be better designed; a review of some of this research is provided in Spagnolo (2008). Starting with the pioneering paper of Motta and Polo (2003), there has been a sequence of theoretical analyses including Spagnolo (2003), Aubert, Kovacic, and Rey (2006), Chen and Rey (2007), Harrington (2008), and Choi and Gerlach (2010). While models and results vary, the overall conclusion is that leniency programs make collusion more difficult.[3] There is also a growing body of experimental work which similarly provides evidence of the efficacy of leniency programs including Apesteguia, Dufwenberg, and Selten (2007), Hinloopen and Soetevent (2008), Dijkstra, Haan, and Schoonbeek (2011), and Bigoni et al (2012). These experimental studies generally find that a leniency program reduces cartel formation though some studies also find that prices are higher, conditional on a cartel forming, when there is a leniency program. Finally, there are an increasing number of empirical studies that measure the impact of leniency programs. Using data over 1985-2005 for the United States, Miller (2009) finds evidence that the 1993 revision reduced the latent cartel rate. In contrast, Brenner (2009) does not find evidence that collusion was made more difficult with

---

[1]Scott D. Hammond, "Cracking Cartels With Leniency Programs," OECD Competition Committee, Paris, France, October 18, 2005.

[2]For a list of countries with leniency programs, see Borrell, Jiménez, and García (2012) who also estimate how leniency programs have changed the perceptions of managers.

[3]There are a variety of effects at work when a leniency program is put in place and some serve to make collusion easier; see Ellis and Wilson (2001) and Chen and Harrington (2007). Generally, these effects net out so that fewer cartels form when there is a leniency program.

the European Commission's 1996 Corporate Leniency Program, though his data is for 1990-2003 and thus does not encompass an important revision in the program in 2002. Preliminary findings in Klein (2010) and Zhou (2011) suggest that the EC's leniency program has been effective.

While the empirical evidence is mixed, the general conclusion from theoretical and experimental research is that leniency programs are effective in shutting down cartels and deterring cartel formation. However, those findings were derived under a crucial but problematic assumption that non-leniency enforcement is unaffected by the introduction of a leniency program. More specifically, it is assumed that the probability that a cartel is discovered, prosecuted, and convicted - in the absence of a firm coming forward under the leniency program - is unchanged with the adoption of a leniency program. Not only is that assumption almost certain to be violated, but conclusions about the efficacy of a leniency program could significantly change once this probability is made endogenous. Let us argue both points.

With the introduction of a leniency program, the investigation of cases not involving leniency is likely to change and, as a result, this will affect the probability that a cartel is caught and convicted. As a competition authority has limited resources, if resources are used to handle leniency cases then fewer resources are available to effectively prosecute non-leniency cases. This doesn't necessarily imply that non-leniency enforcement is weaker, however. If a leniency program is successful in reducing the number of cartels, there will be fewer non-leniency cartel cases, in which case the authority may still have ample resources to effectively prosecute them. Furthermore, an optimizing competition authority is likely to adjust its enforcement policy - for example, how it allocates prosecutorial resources across cases - in response to what is occurring with leniency applications. Thus, while we expect the probability that a cartel is caught and convicted to change when a leniency program is put in place, it isn't clear in which direction it will go.

The next point to note is that a change in the likelihood of getting a conviction for a non-leniency case has implications for the efficacy of the leniency process itself. A cartel member will apply for leniency only if it believes that doing so is better than running the risk of being caught and paying full penalties. Thus, the probability of being caught and convicted is integral to inducing firms to apply for leniency. If this probability is very low then few cartel members will use the leniency program, while if the probability is sufficiently high then, under the right circumstances, a cartel member will apply for leniency. The efficacy of a leniency program is then intrinsically tied to how a leniency program affects the probability of being caught and convicted when no firm applies for leniency.

These issues have been recognized in the policy realm as legitimate concerns. In recent years the Directorate General Competition (DG Comp) of the European Commission has been overwhelmed with leniency applications which limits the amount of resources for prosecuting other cases:[4]

> DG Competition is now in many ways the victim of its own success;
> leniency applicants are flowing through the door of its Rue Joseph II

---

[4]Riley (2007), pp. 1-2.

offices, and as a result the small Cartel Directorate is overwhelmed with work. ... It is open to question whether a Cartel Directorate consisting of only approximately 60 staff is really sufficient for the Commission to tackle the 50 cartels now on its books.

Furthermore, the interaction between adoption of a leniency program and enforcement through means other than leniency applications is emphasized in Friederiszick and Maier-Rigaud (2008). Both authors were members of DG Comp and their paper argues that the DG Comp should be more active in detecting cartels and more generally in initiating cases *because* of the success of the leniency program.

The primary contribution of this paper is to assess the impact of a leniency program on the cartel rate while endogenizing the efficacy of non-leniency enforcement. This analysis is done in the context of a population of industries in which cartels are formed and dissolved either due to internal collapse or the efforts of the competition authority. The focus is on how a leniency program affects the steady-state fraction of industries that are cartelized. Non-leniency enforcement is measured by the probability that a cartel is caught, prosecuted, and convicted without use of the leniency program. The probability of conviction depends on the size of the competition authority's caseload which is composed of both leniency and non-leniency cases. This structure creates a feedback relationship that simultaneously determines the efficacy of a leniency program and non-leniency enforcement: A leniency program affects the number of cartels which influences the competition authority's caseload which influences the rate at which non-leniency cases are won which influences expected penalties which influences cartel formation and expected cartel duration and, therefore, the number of cartels.

Our main findings are the following. Allowing non-leniency enforcement to respond to the introduction of a leniency program can either reinforce the efficacy of a leniency program or work against it, even to the point that there are more cartels after introducing a leniency program. The main driving force is to what extent a leniency program adds to a competition authority's caseload and weakens non-leniency enforcement by effectively crowding out non-leniency cases. Note that crowding out is not necessarily implied by an effective leniency program because it could deter cartel formation to the point that there are few leniency applications. In fact, sufficient conditions are provided for a leniency program to lower the cartel rate while producing no applications. The cartel rate is also lower when leniency cases can be handled with few resources or when non-leniency enforcement was sufficiently weak prior to the introduction of a leniency program. However, when leniency cases are resource-intensive and penalties are weak then a leniency program is not only ineffective but harmful in that the cartel rate is higher. This is true even if the competition authority is choosing its caseload to minimize the cartel rate.

Whether or not a leniency program raises or lowers the cartel rate, it is shown to have a differential effect across industries. For industries that tend to produce relatively unstable cartels, a leniency program makes collusion more difficult as reflected in cartels no longer forming or shorter duration for those that do form. However, in industries that tend to produce relatively stable cartels, a leniency program can actu-

4

ally increase cartel duration. This differential effect arises because relatively unstable cartels are more concerned with a race for a leniency, while relatively stable cartels are more concerned with detection and prosecution through non-leniency means. Thus, a leniency program can weaken the less stable cartels but strengthen the more stable cartels when a leniency program undermines non-leniency enforcement.

In the next section, the model is presented. In Section 3, the conditions determining the equilibrium cartel rate are derived. Existence and some basic properties of the equilibrium cartel rate are established in Section 4. Section 5 provides the central results, while Section 6 concludes. All proofs are in an appendix.

2   Model

The modelling strategy is to build a birth and death Markov process for cartels in order to generate an average cartel rate for a population of industries, and to then assess how the introduction of a leniency program influences the frequency of cartels. We build upon the birth and death process developed in Harrington and Chang (2009) by allowing for a leniency program and, most crucially, endogenizing non-leniency enforcement.[5]

2.1   Industry Environment

Firm behavior is modelled using a modification of a Prisoners' Dilemma formulation. Firms simultaneously decide whether to *collude* (set a high price) or *compete* (set a low price). Prior to making that choice, firms observe a stochastic realization of the market's profitability that is summarized by the variable $\pi \geq 0$.[6] If all firms choose *collude* then each firm earns π, while if all choose *compete* then each earns $\alpha\pi$ where $\alpha \in [0,1)$. $1 - \alpha$ then measures the competitiveness of the non-collusive environment. π has a continuously differentiable cdf $H : [\underline{\pi}, \overline{\pi}] \rightarrow [0,1]$ where $0 < \underline{\pi} < \overline{\pi}$. $h(\cdot)$ denotes the associated density function and let $\mu \equiv \int \pi h(\pi) d\pi$ denote its finite mean. If all other firms choose *collude*, the profit a firm earns by deviating - choosing *compete* - is $\eta\pi$ where $\eta > 1$. This information is summarized in the table below. Note that the Bertrand price game is represented by $(\alpha, \eta) = (0, n)$ where n is the number of firms. The Cournot quantity game with linear demand and cost functions in which firms collude at the joint profit maximum is represented by $(\alpha, \eta) = \left(\frac{4n}{(n+1)^2}, \frac{(n+1)^2}{4n} \right)$.[7]

Own action	All other firms' actions	Own profit
collude	*collude*	π
compete	*collude*	$\eta\pi$
compete	*compete*	$\alpha\pi$

---

[5] The Harrington-Chang birth and death process for cartels is estimated in Hyytinen, Steen, and Toivanen (2010) who use data from the Finnish cartel registry.

[6] The informational setting is as in Rotemberg and Saloner (1986).

[7] We have only specified a firm's profit when all firms choose *compete*, all firms choose *collude*, and it chooses *compete* and all others firm choose *collude*. We must also assume that *compete* strictly dominates *collude* for the stage game. It is unnecessary to provide any further specification.

Firms interact in an infinite horizon setting where $\delta \in (0,1)$ is the common discount factor. It is not a repeated game because, as explained later, each industry is in one of two states: cartel and non-cartel. If *firms are a cartel* then they have the opportunity to collude but do so if and only if (iff) it is incentive compatible. More specifically, if firms are cartelized then they simultaneously choose between *collude* and *compete*, and, at the same time, whether or not to apply to the corporate leniency program. Details on the description of the leniency program are provided later. If it is incentive compatible for all firms to choose *collude* then each earns π. If instead a firm prefers *compete* when all other firms choose *collude* then collusion is not incentive compatible (that is, it is not part of the subgame perfect equilibrium for the infinite horizon game) and each firm earns $\alpha\pi$. In that case, collusion is not achieved. If *firms are not a cartel* then each firm earns $\alpha\pi$ as, according to equilibrium, they all choose *compete*.

At the end of the period, there is the random event whereby the competition authority (CA) may pursue an investigation; this can only occur if firms colluded in the current or previous period and no firm applied for leniency.[8] Let $\sigma \in [0,1]$ denote the probability that firms are discovered, prosecuted, and convicted (below, we will endogenize σ though, from the perspective of an individual industry, it is exogenous). In that event, each firm incurs a penalty of F.

It is desirable to allow F to depend on the extent of collusion. Given there is only one level of collusion in the model, the "extent of collusion" necessarily refers to the number of periods that firms had colluded. A proper accounting of that effect would require that each cartel have a state variable equal to the length of time for which it has been active; such an extension would seriously complicate the analysis. As a simplifying approximation, it is instead assumed that the penalty is proportional to the average increase in profit from being cartelized (rather than the *realized* increase in profit). If Y denotes the expected per period profit from being in the "cartel state" then $F = \gamma (Y - \alpha\mu)$ where $\gamma > 0$ and $\alpha\mu$ is average non-collusive profit. This specification avoids the need for state variables but still allows the penalty to be sensitive to the (average) extent of collusion.[9]

In addition to being discovered by the CA, a cartel can be uncovered because one of its members comes forth under the corporate leniency policy. Suppose a cartel is in place. If a single firm applies for leniency then all firms are convicted for sure and the firm that applied receives a penalty of θF where $\theta \in [0,1)$, while the other cartel members each pay F. If all firms simultaneously apply for leniency then each firm pays a penalty of ωF where $\omega \in (\theta, 1)$. For example, if only one firm can receive leniency and each firm has an equal probability of being first in the door then $\omega = \frac{n-1+\theta}{n}$ when there are n cartel members. It is sufficient for the ensuing analysis that we specify the leniency program when either one firm applies or all firms apply. Also,

---

[8] To allow it to depend on collusion farther back in time would require introducing another state variable that would unnecessarily complicate the analysis. Having it depend on collusion in the previous period will simplify some of the expressions and, furthermore, it seems quite reasonable that detection can occur, to a limited extent, after the fact.

[9] A more standard assumption in the literature is to assume F is fixed which is certainly simpler but less realistic than our specification. All of our qualitative results hold when F is fixed.

leniency is not awarded to firms that apply after another firm has done so.

From the perspective of firms, competition policy is summarized by the four-tuple $(\sigma, \gamma, \theta, \omega)$ which are, respectively, the probability of paying penalties through non-leniency enforcement, the penalty multiple, the leniency parameter when only one firm applies (where $1 - \theta$ is the proportion of fines waived), and the leniency parameter when all firms apply (where $1 - \omega$ is the proportion of fines waived).

Next, let us describe how an industry's cartel status evolves. Suppose it enters the period cartelized. The industry will exit the period still being a cartel if: 1) all firms chose *collude* (which requires that collusion be incentive compatible); 2) no firm applied for leniency; and 3) the CA did not discover and convict the firms of collusion. Otherwise, the cartel collapses and firms revert to the "no cartel" state. If instead the industry entered the period in the "no cartel" state then with probability $\kappa \in (0, 1)$ firms cartelize. For that cartel to still be around at the end of the period, conditions (1)-(3) above must be satisfied. Note that whenever a cartel is shutdown - whether due to internal collapse, applying to the leniency program, or having been successfully prosecuted - the industry may re-cartelize in the future. Specifically, it has an opportunity to do so with probability κ in each period that it is not currently colluding.[10] The timing of events is summarized in the figure below.
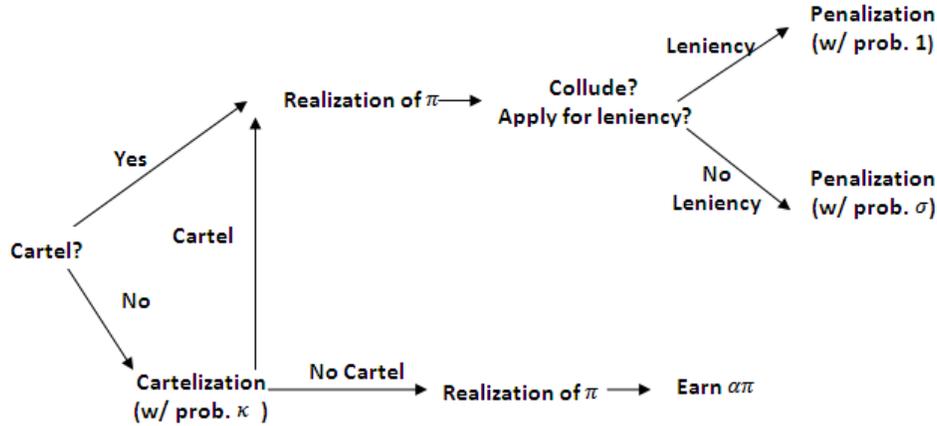
In modelling a population of industries, it is compelling to allow industries to vary in terms of cartel stability. For this purpose, industries are assumed to differ in the parameter η. If one takes this assumption literally, it can be motivated by heterogeneity in the elasticity of firm demand or the number of firms (as with the Bertrand price game). Our intent is not to be literal but rather to think of this as a parsimonious way in which to encompass industry heterogeneity. Let the cdf on industry types be represented by the continuously differentiable strictly increasing

---

[10]Alternatively, one could imagine having two distinct probabilities - one to reconstitute collusion after a firm cheated (the probability of moving from the punishment to the cooperative phase) and another to reform the cartel after having been convicted. For purposes of parsimony, those two probabilities are assumed to be the same.

function $G : [\underline{\eta}, \overline{\eta}] \rightarrow [0, 1]$ where $1 < \underline{\eta} < \overline{\eta}$. $g(\cdot)$ denotes the associated density function. The appeal of η is that it is a parameter which influences the frequency of collusion but does not directly affect the value of the firm's profit stream since, in equilibrium, firms do not cheat; this property simplifies the analysis.

2.2 Enforcement Technology

Non-leniency enforcement is represented by σ which is the probability that a cartel pays penalties without one of its members having entered the leniency program. Here, we explain how σ is determined. σ is the compound probability that: 1) the cartel is discovered by the CA; 2) the CA decides to investigate the cartel; and 3) the CA is successful in its investigation and penalties are levied. The initial discovery of a cartel is presumed to be exogenous and to come from customers, uninvolved employees, the accidental discovery of evidence through a proposed merger, and so forth. $q \in [0, 1]$ denotes the probability of discovery and is a parameter throughout the paper. What the CA controls is how many cases to take on which is represented by $r \in [0, 1]$ which is the fraction of reported cases that the CA chooses to investigate. Initially, we will derive results when r is fixed and then allow r to be endogenous. Finally, of those cases discovered and investigated, the CA is successful in a fraction $s \in [0, 1]$ of them where s is determined by the relationship between the CA's resources and its caseload.[11]

The CA is faced with a resource constraint: the more cases it takes on, the fewer resources are applied to each case and the lower is the probability of winning any individual case. More specifically, it is assumed

$$s = p(\lambda L + R) \text{ where } \lambda \in (0, 1].$$

L is the fraction of industries that are involved in leniency cases, R is the fraction of industries that are involved in non-leniency cases, and s is the proportion of R cases that result in a conviction. Leniency cases are assumed to be won for sure. $\lambda \leq 1$ because leniency cases may take up fewer resources than those cases lacking an informant. We will refer to $L + R$ as the number of cases and $\lambda L + R$ as the caseload. $p : [0, 1] \rightarrow [0, 1]$ is a continuous decreasing function so that a bigger caseload means a lower probability of winning a non-leniency case. In sum, the probability that a cartel pays penalties is

$$\sigma = q \times r \times s = q \times r \times p(\lambda L + R).$$

σ is endogenous because s is determined by the caseload which depends on the number of cartels, and r may be chosen by the CA.[12]

---

[11] In a richer model, we could allow for heterogeneity across cartel cases in terms of the perceived difficulty of gaining a conviction; that is, s is cartel-specific. The CA would then decide not only how many cases to pursue but which cases to pursue.

[12] It should be noted that Motta and Polo (2003) do allow for optimal enforcement expenditure by modelling a trade-off between monitoring and prosecution. They endow a CA with a fixed amount of resources that can be allocated between finding suspected episodes of collusion and prosecuting the

Key to the analysis is the implicit assumption that the CA faces a resource constraint in the sense that resources per case decline with the number of cases as reflected in the specification that the probability of any investigation being successful is decreasing in the caseload. In practice, an CA can move around resources to handle additional cartel activity by, for example, shifting lawyers and economists from merger cases to cartel cases. However, there is a rising opportunity cost in doing so and that ought to imply that resources per cartel case will decline with the number of cartel cases. Of course, CA officials can lobby their superiors (either higher level bureaucrats or elected officials) for a bigger budget but, at least in the U.S., the reality is that the CA's budget does not scale up with its caseload. While the budget of the Antitrust Division of the U.S. Department of Justice is increasing in GDP (Kwoka, 1999), DOJ antitrust case activity is actually countercyclical (Ghosal and Gallo, 2001).

The last element to specify is the determination of the fraction of cases that the CA takes on, r, which requires specifying an objective to the CA. As a benchmark, it is assumed here that the CA is welfare-maximizing in that it chooses r to minimize the cartel rate. The results in Section 5 actually are true either holding r fixed or with this objective assigned to the CA. Some numerical analysis is also conducted and there it is assumed r is selected to minimize the cartel rate because it eliminates having to specify one more parameter value. We view the assumption that the CA minimizes the cartel rate to be a useful benchmark for gaining insight into the possible implications of a leniency program though not necessarily as a good description of CA behavior. However a CA is rewarded, it is natural to assume that rewards are based on observable measures of performance. Given that the cartel rate is not observable (only discovered cartels are observable) then presumably the CA is not rewarded based on how its policies affect the cartel rate (at least not directly). Thus, it is not clear that there is an incentive scheme that will induce the CA to minimize the cartel rate. We intend to explore the modelling of the CA in future research.[13]

3   Equilibrium Conditions

In this section, we describe the conditions determining the equilibrium frequency with which industries are cartelized. Prior to getting into the details, let us provide a brief overview.

1. Taking as given σ (the per period probability that a cartel pays penalties through non-leniency enforcement), we first solve for equilibrium collusive behavior for a type-η industry and the maximum value for π whereby collusion is

---

cases that are found or, in the language of our model, between raising q and lowering s (assuming $r = 1$). However, the model is very different from ours - for example, they do not consider a population of industries and do not solve for the steady-state frequency of cartels - and it does not address the questions we are raising here.

[13] In Harrington (2011a), sufficient conditions are derived for the optimal policy of a CA, whose objective is to maximize the number of (observable) convictions, to coincide with the policy that minimizes the cartel rate.

incentive compatible, denoted $\phi^*\left(\sigma, \eta\right)$.

2. With the conditions for internal collapse - which occurs when $\pi > \phi^*\left(\sigma, \eta\right)$ - and the likelihood of non-leniency enforcement, σ, along with the probability of cartel formation, κ, a Markov process on cartel birth and death is constructed from which is solved the stationary distribution of industries in terms of their cartel status, for each industry type η. By aggregating over all industry types, the equilibrium cartel rate, $C\left(\sigma\right)$, is derived, given σ.

3. The next step is to solve for the equilibrium value of σ, denoted σ^*. The probability that the CA's investigation is successful, $p\left(\lambda L + R\right)$, depends on the mass of leniency cases, L, and the mass of non-leniency cases, R; both L and R depend on σ as they depend on the cartel rate $C\left(\sigma\right)$. σ^* is then a fixed point:

$$\sigma^* = qrp\left(\lambda L\left(\sigma^*\right) + R\left(\sigma^*\right)\right)$$

In other words, σ - the probability that firms are caught, prosecuted, and convicted - determines the cartel rate $C\left(\sigma\right)$, the cartel rate determines the caseload $\lambda L\left(\sigma\right) + R\left(\sigma\right)$, and the caseload determines the probability that they are able to get a conviction on a case and thus σ. Given σ^*, the equilibrium cartel rate is $C\left(\sigma^*\right)$. Section 4 derives some properties of $C\left(\sigma\right)$ and proves the existence of σ^*.

4. When r is fixed, the analysis ends with step 3. When r is endogenous, the final step is to solve for the value that minimizes the cartel rate:

$$r^* \in \arg \min_{r \in [0,1]} C\left(\sigma^*\left(r\right)\right).$$

3.1 Cartel Formation and Collusive Value

A collusive strategy for a type-η industry entails colluding when π is sufficiently low and not colluding otherwise. The logic is as in Rotemberg and Saloner (1986). When π is high, the incentive to deviate is strong because a firm increases current profit by $\left(\eta - 1\right)\pi$. At the same time, the future payoff is independent of the current realization of π, given that π is *iid*. Since the payoff to cheating is increasing in π while the future payoff is independent of π, the incentive compatibility of collusion is more problematic when π is higher.

Suppose firms are able to collude for at least some realizations of π, and let W^o and Y^o denote the payoff when the industry is not cartelized and is cartelized, respectively. If not cartelized then, with probability κ, firms have an opportunity to cartelize with resulting payoff Y^o. With probability $1 - \kappa$, firms do not have such an opportunity and continue to compete. In that case, each firm earns current expected profit of $\alpha\mu$ and a future value of W^o. Thus, the payoff when not colluding is defined recursively by:

$$W^o = \left(1 - \kappa\right)\left(\alpha\mu + \delta W^o\right) + \kappa Y^o. \tag{1}$$

As it'll be easier to work with re-scaled payoffs, define:

$$W \equiv (1 - \delta) W^o, \ Y \equiv (1 - \delta) Y^o.$$

Multiplying both sides of (1) by $1 - \delta$ and re-arranging yields:

$$W = \frac{(1 - \kappa)(1 - \delta)\alpha\mu + \kappa Y}{1 - \delta(1 - \kappa)}$$

Also note that the incremental value to being in the cartelized state is:

$$Y - W = Y - \left(\frac{(1 - \kappa)(1 - \delta)\alpha\mu - \kappa Y}{1 - \delta(1 - \kappa)} \right) = \frac{(1 - \kappa)(1 - \delta)(Y - \alpha\mu)}{1 - \delta(1 - \kappa)}. \quad (2)$$

Suppose firms are cartelized and π is realized. When a firm decides whether to collude or cheat, it decides at the same time whether to apply for leniency. If it decides to collude, it is clearly not optimal to apply for leniency since the cartel is going to be shut down by the authorities in which case the firm ought to maximize current profit by cheating. The more relevant issue is whether it should apply for leniency if it decides to cheat. The incentive compatibility constraint (ICC) is:

$$(1 - \delta)\pi + \delta[(1 - \sigma)Y + \sigma W] - (1 - \delta)\sigma\gamma(Y - \alpha\mu) \quad (3)$$
$$\geq \ (1 - \delta)\eta\pi + \delta W - (1 - \delta)\min\{\sigma, \theta\}\gamma(Y - \alpha\mu).$$

Examining the LHS expression, if it colludes then it earns current profit of π (given all other firms are colluding). With probability $1 - \sigma$, the cartel is not shut down by the CA and, given the industry is in the cartel state, the future payoff is Y. With probability σ, the cartel is caught and convicted by the CA - which means a one-time penalty of $\gamma(Y - \alpha\mu)$ - and since the industry is no longer cartelized, the future payoff is W. Turning to the RHS expression, the current profit from cheating is $\eta\pi$. Since this defection causes the cartel to collapse, the future payoff is W. There is still a chance of being caught and convicted and a deviating firm will apply for leniency iff the penalty from doing so is less than the expected penalty from not doing so (and recall that the other firms are colluding and thus do not apply for leniency); that is, when $\theta\gamma(Y - \alpha\mu) < \sigma\gamma(Y - \alpha\mu)$ or $\theta < \sigma$. Given optimal use of the leniency program, the deviating firm's expected penalty is then $\min\{\sigma, \theta\}\gamma(Y - \alpha\mu)$. Re-arranging (3) and using (2), the ICC can be presented as:

$$\pi \ \leq \ \frac{\delta(1 - \sigma)(1 - \kappa)(Y - \alpha\mu) - [1 - \delta(1 - \kappa)][\sigma - \min\{\sigma, \theta\}]\gamma(Y - \alpha\mu)}{(\eta - 1)[1 - \delta(1 - \kappa)]} \quad (4)$$
$$\equiv \ \phi(Y, \sigma, \eta).$$

Collusion is incentive compatible iff the current market condition is sufficiently low.[14]

---

[14] As specified in the ICC in (3), the penalty is slightly different from that in Harrington and Chang (2009) or HC09. In terms of rescaled payoffs, HC09 assumes the penalty is $\gamma(Y - \alpha\mu)$, while here it

11

In deriving an expression for the value to colluding, we need to discuss usage of the leniency program in equilibrium. Firms do not use it when market conditions result in the cartel being stable but may use it when the cartel collapses. As the continuation payoff is W regardless of whether leniency is used, a firm applies for leniency iff it reduces the expected penalty. First note that an equilibrium either has no firms applying for leniency or all firms doing so because if at least one firm applies then another firm can lower its expected penalty by also doing so. This has the implication that it is always an equilibrium for all firms to apply for leniency. Furthermore, it is the unique equilibrium when $\theta < \sigma$. To see why, suppose all firms were not to apply for leniency. A firm would then lower its penalty from σF to θF by applying. When instead $\sigma \leq \theta$, there is also an equilibrium in which no firm goes for leniency as to do so would increase its expected penalty from σF to θF. Using the selection criterion of Pareto dominance, we will assume that, upon internal collapse of the cartel, no firms apply when $\sigma \leq \theta$ and all firms apply when $\theta < \sigma$.

The expected payoff to being cartelized, $\psi(Y, \sigma, \eta)$, is then recursively defined by:

$$\psi(Y,\sigma,\eta) = \begin{cases} \int_{\underline{\pi}}^{\phi(Y,\sigma,\eta)} \{(1-\delta)\pi + \delta[(1-\sigma)Y + \sigma W] - (1-\delta)\sigma\gamma(Y-\alpha\mu)\} h(\pi)\,d\pi & \text{if } \sigma \leq \theta \\ + \int_{\phi(Y,\sigma,\eta)}^{\overline{\pi}} [(1-\delta)\alpha\pi + \delta W - (1-\delta)\sigma\gamma(Y-\alpha\mu)] h(\pi)\,d\pi & \\ \\ \int_{\underline{\pi}}^{\phi(Y,\sigma,\eta)} \{(1-\delta)\pi + \delta[(1-\sigma)Y + \sigma W] - (1-\delta)\sigma\gamma(Y-\alpha\mu)\} h(\pi)\,d\pi & \text{if } \theta < \sigma \\ + \int_{\phi(Y,\sigma,\eta)}^{\overline{\pi}} [(1-\delta)\alpha\pi + \delta W - (1-\delta)\omega\gamma(Y-\alpha\mu)] h(\pi)\,d\pi & \end{cases}.$$

To understand this expression, first consider when $\sigma \leq \theta$, in which case leniency is not used. If $\pi \in [\underline{\pi}, \phi(Y,\sigma,\eta)]$ then collusion is incentive compatible; each firm earns current profit of π, incurs an expected penalty of $\sigma\gamma(Y-\alpha\mu)$, and has an expected future payoff of $(1-\sigma)Y + \sigma W$. If instead $\pi \in (\phi(Y,\sigma,\eta), \overline{\pi}]$ then collusion is not incentive compatible; so each firm earns current profit of $\alpha\pi$, incurs an expected penalty of $\sigma\gamma(Y-\alpha\mu)$, and has an expected future payoff of W. The expression when $\theta < \sigma$ differs only when collusion breaks down in which case all firms apply for leniency and the expected penalty is $\omega\gamma(Y-\alpha\mu)$.[15]

is $(1-\delta)\gamma(Y-\alpha\mu)$. This means that HC09 assumes that a conviction results in an infinite stream of single-period penalties of $\gamma(Y-\alpha\mu)$ which has a present value of $\gamma\left(\frac{Y-\alpha\mu}{1-\delta}\right)$, while the current paper assumes a one-time penalty of $(1-\delta)\gamma(Y-\alpha\mu)$ which has a present value of $\gamma(Y-\alpha\mu)$. We now believe the latter specification is more sound. For the specification in HC09, every time a cartel is convicted, it has to pay a penalty of $\gamma(Y-\alpha\mu)$ *ad infinitum*. Thus, if it has been convicted k times in the past then it is paying $k\gamma(Y-\alpha\mu)$ in each period, while earning an average collusive profit of μ in each period. As $k \to \infty$, the penalty is unbounded while the payoff from collusion is not. It can be shown that the penalty specification in HC09 implies $\lim_{\delta\to 1} Y = \alpha\mu$ so that the penalty wipes out all gains from colluding. These properties do not seem desirable, and we believe it is better to assume the penalty is a one-time payment $\gamma(Y-\alpha\mu)$ rather than an infinite stream of $\gamma(Y-\alpha\mu)$. It is important to note that this change in specification does not affect the conclusions in HC09 because of the parameter γ. Starting with the original specification $\gamma(Y-\alpha\mu)$ and defining $\widetilde{\gamma} \equiv \gamma/(1-\delta)$, the analysis in HC09 is equivalent to when the penalty is $(1-\delta)\widetilde{\gamma}(Y-\alpha\mu)$. This transformation works as long as δ is fixed. As the main results in HC09 do not involve performing comparative statics with respect to δ or letting $\delta \to 1$, the conclusions in HC09 remain intact.

[15] Note that if market conditions are sufficiently strong - that is, $\pi > \phi(Y,\sigma,\eta)$ - firms not only

A fixed point to ψ is an equilibrium value for Y. That is, given an anticipated future collusive value Y, the resulting equilibrium behavior - as represented by $\phi(Y, \sigma, \eta)$ - results in firms colluding for market states such that the value to being in a cartel is Y. We then want to solve:

$$Y^* = \psi(Y^*, \sigma, \eta).$$

As an initial step to exploring the set of fixed points, first note that $\psi(\alpha\mu, \sigma, \eta) = \alpha\mu$. Hence, one fixed point to ψ is the degenerate solution without collusion. If there is a fixed point with collusion - that is, $Y > \alpha\mu$ - then we select the one with the highest value.

Given $Y^*(\sigma, \eta)$, define

$$\phi^*(\sigma, \eta) \equiv \max\{\min\{\phi(Y^*(\sigma, \eta), \sigma, \eta), \overline{\pi}\}, \underline{\pi}\},$$

as the maximum profit realization such that a type-η cartel is stable. It is a measure of cartel stability since the cartel is stable iff $\pi \leq \phi^*(\sigma, \eta)$ and thus internally collapses with probability $1 - H(\phi^*(\sigma, \eta))$. Note that if $\phi^*(\sigma, \eta) = \overline{\pi}$ then the cartel is stable for all market conditions (so it never internally collapses), and if $\phi^*(\sigma, \eta) = \underline{\pi}$ then the cartel is unstable for all market conditions (so firms never collude).

3.2 Stationary Distribution of Cartels

Given $\phi^*(\sigma, \eta)$, the stochastic process by which cartels are born and die (either through internal collapse or being shut down by the CA) is characterized in this section. The random events driving this process are the opportunity to cartelize, market conditions, and conviction by the CA. We initially characterize the stationary distribution for type-η industries. The stationary distribution for the entire population of industries is then derived by integrating the type specific distributions over all types.

Consider an arbitrary type-η industry. If it is not cartelized at the end of the preceding period then, by the analysis in Section 3.1, it'll be cartelized at the end of the current period with probability $\kappa(1 - \sigma)H(\phi^*(\sigma, \eta))$. With probability κ it has the opportunity to cartelize, with probability $H(\phi^*(\sigma, \eta))$ the realization of π is such that collusion is incentive compatible, and with probability $1 - \sigma$ it is not caught and convicted by the CA. If instead the industry was cartelized at the end of the previous period, it'll still be cartelized at the end of this period with probability $(1 - \sigma)H(\phi^*(\sigma, \eta))$.

---

do not collude (as it is not incentive compatible) but the cartel breaks down, as reflected in firms having a future payoff of W (less expected penalties) An alternative strategy is to have firms not collude when market conditions are strong but for the cartel to remain in place so that firms collude again as soon as market conditions return to lower levels, in which case the future payoff is Y. The latter equilibrium is more in the spirit of the traditional approach to modelling collusive behavior in that the degree of collusion adjusts to market conditions rather than cartel breakdown occurring. We do not characterize such an equilibrium because, in practice, cartels *do* breakdown - it is not simply that firms go to a coordinated punishment - and it is that death process we want our model to generate. In a richer model in which firms could choose from an array of prices, we would be fine with having some adjustment of the collusive price to market conditions - rather than always having cartel breakdown - as long as, under some market conditions, the cartel does collapse.

Let $NC(\sigma, \eta)$ denote the proportion of type-η industries that are not cartelized. The stationary rate of non-cartels is defined by :

$$
\begin{aligned}
NC(\sigma, \eta) &= NC(\sigma, \eta)\left[(1-\kappa) + \kappa(1-H(\phi^*)) + \kappa\sigma H(\phi^*)\right] \\
&+ \left[1 - NC(\sigma, \eta)\right]\left[(1-H(\phi^*)) + \sigma H(\phi^*)\right]
\end{aligned}
\tag{5}
$$

Examining the RHS of (5), a fraction $NC(\sigma, \eta)$ of type-η industries were not cartelized in the previous period. Out of those industries, a fraction $1-\kappa$ will not have the opportunity to cartelize in the current period. A fraction $\kappa(1-H(\phi^*))$ will have the opportunity but, due to a high realization of π, find it is not incentive compatible to collude, while a fraction $\kappa\sigma H(\phi^*)$ will cartelize and collude but then are discovered by the CA. Of the industries that were colluding in the previous period, which have mass $1 - NC(\eta)$, a fraction $1 - H(\phi^*)$ will collapse for internal reasons and a fraction $\sigma H(\phi^*)$ will instead be shut down by the authorities.

Solving (5) for $NC(\sigma, \eta)$:

$$
NC(\sigma, \eta) = \frac{1 - (1-\sigma)H(\phi^*(\sigma, \eta))}{1 - (1-\kappa)(1-\sigma)H(\phi^*(\sigma, \eta))}.
\tag{6}
$$

For the stationary distribution, the fraction of cartels among type-η industries is then:

$$
C(\sigma, \eta) \equiv 1 - NC(\sigma, \eta) = \frac{\kappa(1-\sigma)H(\phi^*(\sigma, \eta))}{1 - (1-\kappa)(1-\sigma)H(\phi^*(\sigma, \eta))}.
\tag{7}
$$

Finally, the derivation of the entire population of industries is performed by integrating the type-η distribution over $\eta \in [\underline{\eta}, \overline{\eta}]$. The mass of cartelized industries, which we refer to as the cartel rate $C(\sigma)$, is then defined by:

$$
C(\sigma) \equiv \int_{\underline{\eta}}^{\overline{\eta}} C(\sigma, \eta) g(\eta) d\eta = \int_{\underline{\eta}}^{\overline{\eta}} \left[\frac{\kappa(1-\sigma)H(\phi^*(\sigma, \eta))}{1 - (1-\kappa)(1-\sigma)H(\phi^*(\sigma, \eta))}\right] g(\eta) d\eta.
\tag{8}
$$

3.3 Equilibrium Non-Leniency Enforcement

Recall that $\sigma = qrs$ where q is the probability of a cartel being discovered, r is the probability that the CA investigates a reported case, and s is the probability of it succeeding with the investigation. We now want to derive the equilibrium value of s, where $s = p(\lambda L + R)$, L is the mass of leniency cases, and R is the mass of non-leniency cases handled by the CA. As both L and R depend on the cartel rate C and the cartel rate depends on s (through σ), this is a fixed point problem. We need to find a value for s, call it s', such that, given $\sigma = qrs'$, the induced cartel rate $C(qrs')$ is such that it generates L and R so that $p(\lambda L + R) = s'$.

With our expression for the cartel rate, we can provide expressions for L and R. The mass of cartel cases generated by the leniency program is:

$$
L(\sigma) = \begin{cases} 0 & \text{if } \sigma \leq \theta \\[2ex] \int_{\underline{\eta}}^{\overline{\eta}} (1 - H(\phi^*(\sigma, \eta))) C(\sigma, \eta) g(\eta) d\eta & \text{if } \theta < \sigma \end{cases}
\tag{9}
$$

14

In (9), note that an industry does not apply for leniency when it is still effectively colluding. When collusion stops, leniency is used when the only equilibrium is that all firms apply for leniency, which is the case when $\theta < \sigma$. Thus, when $\theta < \sigma$, L equals the mass of cartels that collapse due to a high realization of π. This is consistent with a concern expressed by a European Commission official that many leniency applicants are from dying cartels.[16, 17]

The mass of cartel cases generated without use of the leniency program is

$$R\left(\sigma\right) = \begin{cases} qrC\left(\sigma\right) & \text{if } \sigma \leq \theta \\[2ex] qr \int_{\underline{\eta}}^{\overline{\eta}} H\left(\phi^*\left(\sigma,\eta\right)\right) C\left(\sigma,\eta\right) g\left(\eta\right) d\eta & \text{if } \theta < \sigma \end{cases} \tag{10}$$

If the leniency program is never used (which is when $\sigma \leq \theta$), then the mass of cases being handled by the CA is $qrC\left(\sigma\right)$. If instead $\theta < \sigma$, so that dying cartels use the leniency program, then the cartels left to be caught are those which have not collapsed in the current period which is $\int_{\underline{\eta}}^{\overline{\eta}} H\left(\phi^*\left(\sigma,\eta\right)\right) C\left(\sigma,\eta\right) g\left(\eta\right) d\eta$.

The equilibrium probability of a CA successfully getting a cartel to pay penalties (without use of the leniency program) is the solution to the following fixed point problem:

$$\sigma = \Psi\left(\sigma\right) \equiv \begin{cases} qrp\left(qrC\left(\sigma\right)\right) & \text{if } \sigma \leq \theta \\[2ex] qrp\left(\lambda \int_{\underline{\eta}}^{\overline{\eta}} \left(1 - H\left(\phi^*\left(\sigma,\eta\right)\right)\right) C\left(\sigma,\eta\right) g\left(\eta\right) d\eta \right. & \text{if } \theta < \sigma \\ \left. + qr \int_{\underline{\eta}}^{\overline{\eta}} H\left(\phi^*\left(\sigma,\eta\right)\right) C\left(\sigma,\eta\right) g\left(\eta\right) d\eta \right) \end{cases} \tag{11}$$

where we have substituted for L using (9) and R using (10).[18] If there are multiple solutions to (11) then it is assumed the maximal one is selected.[19]

3.4 Optimal Competition Policy

The analytical results in Section 5 are derived taking enforcement policy - as parameterized by r which is the fraction of possible cases that the CA takes on - as fixed. However, as argued later, they extend as well to when r is endogenized by assuming the CA acts to minimize the cartel rate. Let $\sigma^*\left(r\right)$ denote the maximal solution to (11), where its dependence on r is now made explicit. For when r is endogenized, it

---

[16] This statement was made by Olivier Guersent at the 11th Annual EU Competition Law and Policy Workshop: Enforcement of Prohibition of Cartels in Florence, Italy in June 2006.

[17] That either all firms or no firms apply for leniency is a property of not only our analysis but all previous analyses on leniency programs with the exception of some recent work by one of the authors, Harrington (2011b, 2012). In those two papers, there is private information between cartel members which can explain why only one firm would come forward to the CA.

[18] Note that the fixed point can be defined in terms of either σ or s given that $\sigma = qrs$ and, at this point of the analysis, q and r are parameters.

[19] We conjecture that results hold with some other selections, such as the minimal fixed point to Ψ. What is necessary is that a shift up (down) in Ψ increases (decreases) σ^*.

is assumed that $r = r^*$ where

$$r^* \in \arg \min_{r \in [0,1]} C(\sigma^*(r)).$$

By having the prosecution policy chosen to minimize the cartel rate, the analysis delivers an upper bound on welfare.

3.5 Summary of Solution Algorithm

1. Given σ and Y and for each η, solve for the maximum market condition (or threshold) for which the ICC is satisfied, $\phi(Y, \sigma, \eta)$.

2. Given σ and for each η, solve for the equilibrium collusive value $Y^*(\sigma, \eta)$ which is a solution to the fixed point problem: $Y^* = \psi(Y^*, \sigma, \eta)$. If there are multiple fixed points, select the maximum. Given $Y^*(\sigma, \eta)$, define the equilibrium threshold $\phi^*(\sigma, \eta)$.

3. Given σ and $\phi^*(\sigma, \eta)$, derive the stationary proportion of type-η industries that are cartels:
$$C(\sigma, \eta) = \frac{\kappa(1 - \sigma) H(\phi^*(\sigma, \eta))}{1 - (1 - \kappa)(1 - \sigma) H(\phi^*(\sigma, \eta))},$$
and integrate over industry types to derive the stationary cartel rate:
$$C(\sigma) = \int_{\underline{\eta}}^{\overline{\eta}} C(\sigma, \eta) g(\eta) \, d\eta.$$

4. Solve for the equilibrium probability of paying penalties through non-leniency enforcement σ^* which is a solution to the fixed point problem: $\sigma^* = \Psi(\sigma^*)$. If there are multiple fixed points, select the maximum. The equilibrium cartel rate is $C(\sigma^*)$.

5. (Optional) Solve for the optimal prosecution rate $r^* : r^* \in \arg \min_{r \in [0,1]} C(\sigma^*(r))$.

4 Equilibrium Cartel Rate: Existence and Properties

Section 4.1 derives some properties of the cartel rate $C(\sigma)$ function. In particular, it is shown that $C(\sigma)$ is decreasing in σ. Taking σ as exogenous, if firms assign a higher probability to the CA discovering, prosecuting, and convicting cartels then a smaller fraction of industries are cartelized, either because fewer cartels form and/or those cartels that form have shorter average duration.[20] In Section 4.2, it is shown that a solution to (11) exists for when there is no leniency program ($\theta = 1$) and there is a full leniency program ($\theta = 0$). While these results are of intrinsic interest, their primary purpose is to provide the foundation for the analysis in Section 5 which explores the impact of a leniency program.

---

[20] The results in Section 4.1 correspond to some properties proven in Theorems 1, 3, 4, and 6 of Harrington and Chang (2009). However, the results in Harrington and Chang (2009) assume σ is sufficiently small which we do not want to do here given that σ is now endogenized. Instead, results are derived for when the penalty multiple γ is sufficiently small.

4.1 Properties of the Cartel Rate Function

The main result of this sub-section is that the cartel rate is decreasing in the probability that cartels assign to paying penalties through non-leniency enforcement, σ. Before launching into the analysis, let us provide an overview. Recall that the value to being in the cartel state is a fixed point: $Y^* = \psi(Y^*)$. Lemma 1 shows that ψ maps $[\alpha\mu, \mu]$ into itself and, given that ψ is a continuous function of Y, a fixed point exists which is the equilibrium collusive value. Recall that $\phi(Y, \sigma, \eta)$ is the maximal market condition whereby collusion is stable; that is, the ICC is satisfied iff $\pi \leq \phi(Y, \sigma, \eta)$. $\phi^*(\sigma, \eta)$ is the equilibrium threshold after substituting in the equilibrium collusive value. Lemma 2 show that Y^* and ϕ^* are decreasing in η and σ so that more intense non-leniency enforcement (that is, a higher probability of paying penalties) lowers the collusive value and makes a cartel less stable (as the cartel collapses with lower market conditions). Lemma 3 shows that the maximal industry type for which a successful cartel can form, $\widehat{\eta}$, is lower when σ is higher. ($\widehat{\eta}$ is that value such that $\phi^*(\sigma, \eta) > \underline{\pi}$ iff $\eta \leq \widehat{\eta}$.) It is then proven using Lemmas 2 and 3 that $C(\sigma)$ is decreasing in σ (Theorem 4).

Results are derived for when the penalty multiple γ is not too high, which must be the case if collusion is to emerge in equilibrium. Lemma 1 considers properties of the collusive value function $\psi(Y)$. Given that the penalty if convicted is $\gamma(Y - \alpha\mu)$ and thus is proportional to the collusive value, if γ is too high then $\psi(Y)$ will have the pathological property that it is *decreasing* in Y; that is, a higher future collusive value Y actually reduces the value to being in the cartel state because it raises the penalty even more. In that case, collusion will not occur. γ must then be sufficiently low so that ψ is increasing in Y and thus there can exist a fixed point exceeding $\alpha\mu$ which means that firms collude with positive probability.

Lemma 1 $\exists \widehat{\gamma} > 0$ *such that if* $\gamma \in [0, \widehat{\gamma})$ *then: (i)* $\psi : [\alpha\mu, \mu] \rightarrow [\alpha\mu, \mu]$; *and (ii)* $\psi'(Y) > 0$, *for all* $Y \in [\alpha\mu, \mu]$.

By Lemma 1, $Y^*(\sigma, \eta)$ exists which is the recursively defined value to a firm in a type-η industry when it is in the cartel state. Lemma 2 shows that this collusive value is higher when non-leniency enforcement is weaker (that is, σ is lower) and the profit gain to cheating is less (η is lower). Recall that collusion internally collapses if and only if $\pi > \phi^*(\sigma, \eta)$. Lemma 2 also shows that a cartel is more stable - in the sense that $\phi^*(\sigma, \eta)$ is higher so it takes more extreme market conditions to violate the ICC - when σ and η are lower.

Lemma 2 $\exists \widehat{\gamma} > 0$ *such that if* $\gamma \in [0, \widehat{\gamma})$ *then: i)* $Y^*(\sigma, \eta)$ *and* $\phi^*(\sigma, \eta)$ *are non-increasing in* σ *and* η; *ii) if* $Y^*(\sigma, \eta) > \alpha\mu$ *then* $Y^*(\sigma, \eta)$ *is decreasing in* σ; *and iii) if* $\phi^*(\sigma, \eta) \in (\underline{\pi}, \overline{\pi})$ *then* $Y^*(\sigma, \eta)$ *is decreasing in* η *and* $\phi^*(\sigma, \eta)$ *is decreasing in* σ *and* η.

Given that $\phi^*(\sigma, \eta)$ is decreasing in η when $\phi^*(\sigma, \eta) \in (\underline{\pi}, \overline{\pi})$ then, if $\phi^*(\sigma, \overline{\eta}) = \underline{\pi}$, there exists $\widehat{\eta}(\sigma) \in [\underline{\eta}, \overline{\eta})$ such that $\phi^*(\sigma, \eta) > \underline{\pi}$ iff $\eta \leq \widehat{\eta}(\sigma)$. Hence, if $\eta \leq \widehat{\eta}(\sigma)$

then an industry can successfully collude with some probability (that is, for some market conditions) and if $\eta > \widehat{\eta}(\sigma)$ then an industry is never able to successfully collude. The next lemma shows that the set of industry types that can successfully collude, $\left[\underline{\eta}, \widehat{\eta}(\sigma)\right]$, shrinks as non-leniency enforcement intensifies (σ is increased).

Lemma 3 $\exists \widehat{\gamma} > 0$ *such that if* $\gamma \in [0, \widehat{\gamma})$ *and* $\widehat{\eta}(\sigma) \in \left(\underline{\eta}, \overline{\eta}\right)$ *then* $\widehat{\eta}(\sigma)$ *is decreasing in* σ.

Theorem 4 shows that more intense non-leniency enforcement reduces the cartel rate. The decline in the cartel rate comes from those cartels that form having shorter duration - because $\phi^*(\sigma, \eta)$ declines (by Lemma 2) - and that fewer industries cartelize - because $\widehat{\eta}(\sigma)$ declines (by Lemma 3).

Theorem 4 $\exists \widehat{\gamma} > 0$ *such that if* $\gamma \in [0, \widehat{\gamma})$ *then* $C(\sigma)$ *is non-increasing in* σ *and if* $C(\sigma) > 0$ *then* $C(\sigma)$ *is decreasing in* σ.

4.2 Existence of an Equilibrium Cartel Rate

From Section 3.3, the equilibrium level of non-leniency enforcement σ^* is the fixed point to (11) which is repeated here:

$$\sigma = \Psi(\sigma) \equiv \begin{cases} qrp\left(qr \int_{\underline{\eta}}^{\overline{\eta}} C(\sigma, \eta) g(\eta) d\eta\right) & \text{if } \sigma \leq \theta \\ \\ qrp\left(\lambda \int_{\underline{\eta}}^{\overline{\eta}} (1 - H(\phi^*(\sigma, \eta))) C(\sigma, \eta) g(\eta) d\eta \right. & \text{if } \theta \leq \sigma \\ \left. + qr \int_{\underline{\eta}}^{\overline{\eta}} H(\phi^*(\sigma, \eta)) C(\sigma, \eta) g(\eta) d\eta\right) \end{cases}$$

where

$$C(\sigma, \eta) = \frac{\kappa(1 - \sigma) H(\phi^*(\sigma, \eta))}{1 - (1 - \kappa)(1 - \sigma) H(\phi^*(\sigma, \eta))}.$$

A fixed point σ^* has the property that if firms believe that the per period probability of paying penalties (through non-leniency enforcement) is σ^* then the induced cartel birth and death rates generate a caseload for the CA whereby the equilibrium conviction rate s^* satisfies $qrs^* = \sigma^*$.

While Ψ maps $[0, 1]$ into itself, the existence of σ^* is not immediate due to two possible sources of discontinuity in Ψ. Recall that $\phi^*(\sigma, \eta)$ depends on $Y^*(\sigma, \eta)$ which is the maximal fixed point to: $Y = \psi(Y, \sigma, \eta)$. Because of multiple fixed points to $\psi(Y, \sigma, \eta)$, $Y^*(\sigma, \eta)$ need not be continuous in σ and if $Y^*(\sigma, \eta)$ is discontinuous then $\phi^*(\sigma, \eta)$ is discontinuous which implies $H(\phi^*(\sigma, \eta))$ and $C(\sigma, \eta)$ are discontinuous. It is proven in Theorem 5 that these possible discontinuities in the integrand of Ψ do not create discontinuities in Ψ. There is a second possible source of discontinuity in Ψ which is due to a discontinuity in expected penalties at $\sigma = \theta$. That discontinuity is present as long as $\theta \in (0, 1)$ and, as a result, existence is proven only when there is no leniency ($\theta = 1$) and full leniency ($\theta = 0$).

Theorem 5 *For* $\theta \in \{0, 1\}$, $\exists \widehat{\gamma} > 0$ *such that if* $\gamma \in [0, \widehat{\gamma})$ *then* σ^* *exists.*

In the ensuing analysis, it is assumed (without being stated) that γ is sufficiently low so that the results of Section 4 apply.

5   Impact of a Leniency Program

In this section, we begin to explore the impact of introducing a corporate leniency program. Does a leniency program always promote desistance (encouraging cartels to shut down) and deterrence (discouraging cartels from forming) or can it be counter-productive and actually result in more cartels? If it can, what are the conditions that avoid such dysfunctional implications and instead ensure that a leniency program reduces the frequency of cartels? What policies can a CA pursue to promote such an outcome?

The analysis will focus on comparing the cases of no leniency program ($\theta = 1$) with a leniency program in which the first firm to come forward receives full leniency ($\theta = 0$). There is no reason to think that results do not extend to when leniency is almost full ($\theta \simeq 0$) though existence of equilibrium has not been established (see the discussion in Section 4.2). To economize on notation and make it easier for the reader to follow the analysis, expressions with an NL subscript will refer to the case of "no leniency program," while those with an L subscript will refer to the case of a "full leniency program." For example, $C_{NL}(\sigma)$ and $C_L(\sigma)$ are, respectively, the cartel rate functions without leniency and with (full) leniency. The associated fixed points for σ are given by:

$$
\sigma_{NL}^* \;=\; qrp\left(qr\int_{\underline{\eta}}^{\overline{\eta}} C_{NL}\left(\sigma_{NL}^*,\eta\right) g\left(\eta\right) d\eta\right)
$$

$$
\sigma_L^* \;=\; qrp\left(\lambda\int_{\underline{\eta}}^{\overline{\eta}}\left(1 - H\left(\phi_L^*\left(\sigma_L^*,\eta\right)\right)\right) C_L\left(\sigma_L^*,\eta\right) g\left(\eta\right) d\eta + qr\int_{\underline{\eta}}^{\overline{\eta}} H\left(\phi_L^*\left(\sigma_L^*,\eta\right)\right) C_L\left(\sigma_L^*,\eta\right) g\left(\eta\right) d\eta\right),
$$

and the associated equilibrium cartel rates are $C_{NL}\left(\sigma_{NL}^*\right)$ and $C_L\left(\sigma_L^*\right)$.

The analysis will also be conducted holding fixed the CA's non-leniency enforcement instrument r, which is the proportion of discovered cases that it prosecutes. Note that if it is shown that a leniency program decreases (increases) the cartel rate for all values of $r > 0$ then allowing r to be chosen to minimize the cartel rate will still result in a leniency program decreasing (increasing) the cartel rate. Thus, the conclusions of this section are applicable to when a CA acts in a welfare-maximizing manner by choosing its prosecutorial caseload to minimize the cartel rate.

Before taking account of how a leniency program endogenously influences non-leniency enforcement (as measured by σ), Section 5.1 shows, under rather general conditions, that a leniency program lowers the cartel rate, holding non-leniency enforcement fixed. This result is of intrinsic interest but is also instrumental for the ensuing analysis. Non-leniency enforcement is then endogenized and, in Section 5.2, sufficient conditions are provided for a leniency program to lower the cartel rate and, in Section 5.3, for a leniency program to raise the cartel rate. Utilizing those results, Section 5.4 draws some general intuition regarding what is required for a leniency

program to serve its intended goal of reducing the cartel rate. Finally, Section 5.5 explores the differential impact of a leniency program across industries.

5.1   Impact of a Leniency Program on the Cartel Rate Function

We begin by analyzing how the cartel rate responds to a leniency program while making the standard assumption in the literature that non-leniency enforcement is exogenous and fixed. Theorem 6 shows that if the probability of paying penalties through non-leniency enforcement is neither too low nor too high - $\sigma \in (\theta, \omega)$ - then a leniency program does not raise the cartel rate. (Recall that a firm pays a fraction θ of the standard penalty when it receives leniency and pays, in expectation, a fraction ω when all firms apply for leniency.) Furthermore, a leniency program strictly reduces the cartel rate if it is further assumed that, without a leniency program, there is a positive measure of industries that cannot fully collude and a positive measure that can collude. This assumption is denoted A1.[21]

Assumption A1 There is positive measure of values for η such that $\phi_{NL}^{*}(\sigma, \eta) < \overline{\pi}$ and a positive measure of values for η such that $\phi_{NL}^{*}(\sigma, \eta) > \underline{\pi}$.

Theorem 6 is derived for the general case when leniency may be partial. For when there is a leniency program, variables have subscript θ to indicate that the policy is that the leniency recipient has a fraction $1 - \theta$ of penalties waived; for example, $C_\theta(\sigma)$ is the cartel rate function in that case.

Theorem 6 If $\sigma \in (\theta, \omega)$ then $C_{NL}(\sigma) \geq C_\theta(\sigma)$ and if A1 holds then $C_{NL}(\sigma) > C_\theta(\sigma)$.

Prior to discussing Theorem 6, let us interpret the restriction that $\sigma \in (\theta, \omega)$. First note that if $\sigma > \theta$ then a firm that contemplates deviating from a cartel would apply for leniency in that instance, as doing so reduces its expected penalty from $\sigma\gamma(Y - \alpha\mu)$ to $\theta\gamma(Y - \alpha\mu)$. $\sigma > \theta$ also has the implication that, in response to the internal collapse of a cartel, all firms apply for leniency because it is not an equilibrium for all firms not to apply. In that case, if $\sigma < \omega$ then a firm's expected penalty rises with a leniency program from $\sigma\gamma(Y - \alpha\mu)$ to $\omega\gamma(Y - \alpha\mu)$. Thus, if $\sigma \in (\theta, \omega)$ then a firm will use the leniency program if it deviates or if the cartel collapses and, in the latter situation, expected penalties are higher compared to when there is no leniency program. In terms of the restrictiveness of $\sigma \in (\theta, \omega)$, if there is full leniency ($\theta = 0$) then $\sigma > \theta$ is not restrictive at all; in fact, we will largely focus our attention on the impact of full leniency. If all firms applying for leniency gives each an equal chance of receiving it, then $\omega = \frac{n-1+\theta}{n} = \frac{n-1}{n} \geq \frac{1}{2}$. Thus, $\sigma < 1/2$ is sufficient for $\sigma < \omega$ to

---

[21]This condition ensures that the cartel rate is positive but not maximal, and rules out case in which a leniency program does not lower the cartel rate because the cartel rate is either zero without a leniency program or the environment is so conducive to collusion that the cartel rate is maximal with or without a leniency program.

be satisfied. For reasonable parameter values, one would expect $\sigma < 1/2$ for, if that is not the case, then it is unlikely that collusion will be profitable.[22]

Under fairly general conditions, Theorem 6 shows that a leniency program reduces the frequency of cartels when non-leniency enforcement is held fixed. Let us summarize the forces that lie behind that result.[23] Consider the matter for an individual cartel of type η. It was shown in the proof of Theorem 6 that $\phi_{NL}(Y, \sigma, \eta) > \phi_\theta(Y, \sigma, \eta)$ so that cartels destabilize for a wider set of conditions with a leniency program. This result is due to the enhanced incentive for a firm to deviate from the cartel. A leniency program increases the payoff to cheating because now a firm can reduce its penalty by simultaneously applying for leniency. This tightens the ICC and (weakly) shrinks the set of market conditions for which collusion is stable from $\pi \in [\underline{\pi}, \phi_{NL}(Y, \sigma, \eta)]$ to $\pi \in [\underline{\pi}, \phi_\theta(Y, \sigma, \eta)]$ which then reduces expected cartel duration and the collusive value function. There is a second effect to a leniency program on the collusive value function which comes when the cartel collapses. If $\theta < \sigma$ then the only equilibrium is for firms to all apply for leniency in which case expected penalties go from $\sigma\gamma(Y - \alpha\mu)$ to $\omega\gamma(Y - \alpha\mu)$. If $\omega > \sigma$ then a leniency program increases expected penalties and this decreases the collusive value function. Hence, a leniency program reduces the collusive value function - $\psi_{NL}(Y, \sigma, \eta) > \psi_\theta(Y, \sigma, \eta)$ - both because it reduces cartel duration and raises penalties in the event of cartel collapse. As a result, equilibrium collusive value is lower - $Y_{NL}^*(\sigma, \eta) > Y_\theta^*(\sigma, \eta)$ - and the equilibrium threshold is lower - $\phi_{NL}^*(\sigma, \eta) > \phi_\theta^*(\sigma, \eta)$. Hence, either a cartel no longer forms - when $\phi_{NL}^*(\sigma, \eta) > \phi_\theta^*(\sigma, \eta) = \underline{\pi}$ - or has shorter duration - when $\phi_{NL}^*(\sigma, \eta) > \phi_\theta^*(\sigma, \eta) > \underline{\pi}$ and, therefore, the cartel rate is lower. Critical to this finding is that a leniency program does not produce leniency applications to add to a CA's caseload.

Taking non-leniency enforcement as exogenous, Theorem 6 tells says that a leniency program can generally be expected to reduce the cartel rate. This finding is consistent with that found in the previous theoretical literature. In the remainder of Section 5, we explore what happens when non-leniency enforcement is endogenous and reacts to the introduction of a leniency program.

5.2 Leniency Program Decreases the Cartel Rate

Recall that an industry has a realization of market condition π in each period where π is a firm's profit when all firms collude, $\alpha\pi$ is a firm's profit when all firms compete, and $\eta\pi$ is a firm's profit when it competes (or cheats) and all rivals collude. A higher value of π makes it more difficult to satisfy the ICC and collusion is stable if and only if π is sufficiently small. π has cdf H with support $[\underline{\pi}, \overline{\pi}]$.

Our first result assumes H is a uniform distribution which is shown in Theorem 7 to have the implication that a cartel is either stable for all $\pi \in [\underline{\pi}, \overline{\pi}]$ or is unstable

---

[22]That conjecture is confirmed by all numerical simualtions conducted which find that, in equilibrium, $\sigma^* << \omega$. Details are in Appendix C.

[23]These forces are not new to the literature; see Motta and Polo (2003), Spagnolo (2003), and Harrington (2008).

for all $\pi \in [\underline{\pi}, \overline{\pi}]$. In that case, a leniency program is assured of reducing the cartel rate even when non-leniency enforcement is endogenous.

Theorem 7 *If H is the uniform distribution and the equilibrium cartel rate without a leniency program is positive - $C_{NL}(\sigma_{NL}^*) > 0$ - then a full leniency program reduces the cartel rate: $C_L(\sigma_L^*) < C_{NL}(\sigma_{NL}^*)$.*

Since a cartel is either fully stable or not stable at all then cartels never internally collapse; they terminate only by being discovered and convicted by the CA. Given that only collapsed cartels apply for leniency, there are then no leniency applications when market conditions are uniformly distributed. Nevertheless, the leniency program promotes cartel deterrence and thereby reduces the cartel rate. The presence of a leniency program enhances the payoff to a firm from cheating because it can set a low price, earn high profit, *and* avoid penalties by going for leniency. As a result, the ICC tightens so that some industries now are no longer able to successfully collude. Holding non-leniency enforcement (that is, σ) fixed, a leniency program then lowers the cartel rate. Allowing non-leniency enforcement to respond reinforces the deterrence effect because the lower cartel rate means fewer non-leniency cases (and there are no leniency cases to add to the caseload) which raises the probability of gaining a conviction and thus raises σ. A higher σ then lowers the collusive value which results in more industries being unable to collude. The endogeniety of non-leniency enforcement to a leniency program then serves to reinforce the efficacy of a leniency program: A leniency program reduces the cartel rate which reduces the CA's caseload which raises non-leniency enforcement which lowers the collusive value which further lowers the cartel rate.

The next result shows that a leniency program reduces the cartel rate when $\sigma_{NL}^* \in (0, \omega)$ and λ is sufficiently low. Recall that if it takes x resources to prosecute a non-leniency case then it takes only λx resources to prosecute a leniency case where $\lambda \in (0, 1]$. If λ is sufficiently low then leniency cases take up few resources compared to non-leniency cases.

Theorem 8 *If $\sigma_{NL}^* \in (0, \omega)$ then there exists $\widehat{\lambda} > 0$ such that if $\lambda \in \left[0, \widehat{\lambda}\right]$ then the equilibrium cartel rate is weakly lower with a leniency program and if A1 holds then the equilibrium cartel rate is strictly lower with a leniency program.*

If there are no leniency applications, a lower cartel rate from having a leniency program means a lower caseload which results in a higher probability of conviction which further reduces the cartel rate. Thus, if the cartel rate is lower given σ then, once σ is endogenized, the equilibrium value of σ is higher and thus the equilibrium cartel rate is even lower. That analysis can change once taking into account leniency applications which contribute to caseload and can reduce σ. However, if λ is sufficiently small, leniency applications do not contribute much to caseload in which case the σ-decreasing force coming from leniency applications adding to caseload is dominated by the σ-increasing force coming from fewer non-leniency cases because

of fewer cartels. Thus, $\sigma_L^* > \sigma_{NL}^*$ and, given that $C_{NL}(\sigma) > C_L(\sigma)$, it follows that the equilibrium cartel rate is lower with a leniency program: $C_{NL}(\sigma_{NL}^*) > C_L(\sigma_L^*)$.

The final result of this sub-section considers when non-leniency enforcement is weak because the likelihood of discovering a cartel is small: $q \simeq 0$. In that case, a leniency program is sure to be beneficial. Theorem 6 suggests that a necessary condition for a leniency program to fail to lower the cartel rate is that it adversely affects non-leniency enforcement. If non-leniency enforcement is absent prior to the introduction of a leniency program then a leniency program cannot further weaken non-leniency enforcement and thus a leniency program must lower the cartel rate. Hence, if a CA is not actively engage in enforcement, a leniency program is sure to be effective in reducing the frequency of cartels.

Theorem 9 *There exists $\widehat{q} > 0$ such that if $q \in [0, \widehat{q}]$ then the equilibrium cartel rate is weakly lower with a leniency program and if A1 holds then the equilibrium cartel rate is strictly lower with a leniency program.*

Theorem 9 is dependent on there being full leniency for the first firm: $\theta = 0$. If $\theta > 0$ then, as $q \to 0$ and $\sigma \to 0$, the leniency program has no effect because a deviator would not use it, and cartel members do not use it upon collapse of the cartel because $\omega > 0$. This comment highlights the complementarity between leniency and non-leniency enforcement; if $\sigma < \theta$ then a leniency program is irrelevant because the chances of being caught through non-leniency means is sufficiently low to make applying for leniency not to be in a firm's interests. Except when leniency is literally full, the efficacy of a leniency program depends on cartel members believing there is at least some chance of them being caught and convicted by the CA.[24]

5.3   Leniency Program Increases the Cartel Rate

The next result shows that a leniency program can actually cause there to be more cartels. Sufficient conditions for this to occur are that penalties are sufficiently weak and a leniency case uses up about as much resources as a non-leniency case. A leniency program is counter-productive because leniency cases crowd out non-leniency cases at the CA which reduces desistance. On the other hand, cartels that die are now assured of paying penalties because one of them is an informant through the leniency program. That has the potential to enhance deterrence and thereby lower the cartel rate, but the effect is small when penalties are weak. In spite of the leniency program apparently "working" in the sense of bringing forth leniency applications, it is actually counter-productive in that the latent cartel rate is higher.[25]

---

[24] An important caveat here is that we have assumed that firms achieve the Pareto-superior equilibrium when it comes to applying for leniency; that is, if there is an equilibrium in which no firms seek leniency then that is the equilibrium upon which firms coordinate. However, experimental evidence suggests that a leniency program can be effective even when $\sigma = 0$ (Bigoni et al, 2012). In that case, presumably a firm is applying for leniency out of concern that a rival will apply for leniency which is sensible for the rival only if it possesses a similar concern.

[25] Theorem 10 is a generic result because it requires that, in an ε-ball around $\gamma = 0$, $Y_{NL}^*(\sigma, \eta)$ and $Y_L^*(\sigma, \eta)$ are continuous in γ.

Theorem 10 *Assume*

$$\int_{\underline{\eta}}^{\overline{\eta}} \left(1 - H\left(\phi_{NL}^*\left(\sigma^*, \eta\right)\right)\right) C_{NL}\left(\sigma_{NL}^*, \eta\right) g\left(\eta\right) d\eta > 0 \qquad (12)$$

so that, without a leniency program, there are cartels that collude and internally collapse. Generically, there exists $\widehat{\lambda} < 1$ and $\widehat{\gamma} > 0$ such that if $(\gamma, \lambda) \in [0, \widehat{\gamma}] \times \left[\widehat{\lambda}, 1\right]$ then the cartel rate with a leniency program strictly exceeds the cartel rate without a leniency program.

If penalties are low then a leniency program can cause the cartel rate to rise because deterrence is not enhanced while desistance is weakened through reduced efficacy in prosecuting non-leniency cases. The introduction of a leniency program causes scarce CA resources to be used on cartels that have already shut down rather than used to convict (and thereby shut down) active cartels that were discovered through non-leniency devices. (12) ensures that, without a leniency program, there are indeed dying cartels so that this crowding out of non-leniency cases does occur. Thus, a leniency program is actually reducing desistance.[26] The potential deterrent role that a leniency program can play here is that it guarantees that dying cartels pay penalties (other than the firm that receives leniency); in comparison, dying cartels incur penalties only with probability σ_{NL}^* when there is no leniency program. However, if penalties are low then the rise in deterrence is small compared to the fall in desistance and, as a result, more cartels form and they last longer which contributes to a higher cartel rate.[27]

Theorem 10 shows that, for any value of r (which is the fraction of possible cases that the CA chooses to prosecute), the cartel rate is higher without a leniency program when penalties are sufficiently weak and a leniency case uses almost as much resources as a non-leniency case to prosecute:[28]

$$C_L\left(qrs_L^*\left(r\right)\right) > C_{NL}\left(qrs_{NL}^*\left(r\right)\right), \ \forall r > 0. \qquad (13)$$

Recall that $\sigma = qrs$ and the dependence of the conviction rate s on r has been made explicit. If the CA chooses its caseload in order to minimize the cartel rate, the optimal enforcement policy with and without a leniency program is:

$$r_{NL}^* \ \in \ \arg\min_{r \in [0,1]} C_{NL}\left(qrs_{NL}^*\left(r\right)\right)$$

$$r_L^* \ \in \ \arg\min_{r \in [0,1]} C_L\left(qrs_L^*\left(r\right)\right).$$

---

[26] Key to reduced desistance is that prosecuting a leniency case does not take up significantly fewer CA resources than a non-leniency case. Prior to becoming Chief Economist of the European Commission, Kai-Uwe Kühn expressed at the Searle Research Symposium in September 2010 that he believed λ is indeed close to one on the basis that leniency cases seem to be as long and involved as non-leniency cases.

[27] Note that (12) rules out the case in which H is a uniform distribution; thus, Theorem 9 does not conflict with Theorem 7.

[28] It does require that for any value of r, (12) is satisfied which is not very restrictive because whether a cartel collapses is partly due to forces unrelated to the CA.

It then follows from (13),

$$C_L \left(q r_L^* s_L^* \left(r_L^* \right) \right) > C_{NL} \left(q r_{NL}^* s_{NL}^* \left(r_{NL}^* \right) \right).$$

Thus the cartel rate is higher with a leniency program even with a welfare-maximizing CA.[29]

5.4 Discussion

Under fairly general conditions, Theorem 6 showed that a leniency program lowers the cartel rate when holding fixed non-leniency enforcement: $C_{NL} \left(\sigma \right) > C_L \left(\sigma \right)$. Whether a leniency program raises or lowers the presence of cartels then comes down to its impact on non-leniency enforcement. If a leniency program strengthens non-leniency enforcement - $\sigma_L^* > \sigma_{NL}^*$ - then clearly a leniency program lowers the cartel rate: $C_L \left(\sigma_L^* \right) < C_{NL} \left(\sigma_{NL}^* \right)$. If a leniency program weakens non-leniency enforcement - $\sigma_L^* < \sigma_{NL}^*$ - then the ultimate impact on the cartel rate depends on the extent to which a leniency program reduces non-leniency enforcement.

The reason that a leniency program can reduce non-leniency enforcement is because it adds to caseload and effectively shifts CA resources away from non-leniency cases. If this crowding out of non-leniency cases is small, a leniency program will not adversely affect non-leniency enforcement in a significant way in which case the cartel rate is lower. Crowding out is small when there are few leniency applications (Theorem 7) or leniency cases can be handled with few resources (Theorem 8) or there were few non-leniency cases in the pre-leniency program environment to now be crowded out (Theorem 9). Alternatively, crowding out can be large when there are many leniency applications and leniency cases are resource-intensive to process. But even in that case, the leniency program can still deter cartel formation or result in less stable cartels as a result of raising expected penalties and thereby strengthen non-leniency enforcement. The size of that effect is directly proportional to the size of penalties in the event of conviction; in this way, a leniency program and penalties are complements in fighting cartels. If penalties are weak then a leniency program will not have significant deterrence effect to offset the crowding out of non-leniency cases in which case the cartel rate rises (Theorem 10).

While these findings were derived as a limiting results - for example, a leniency program raises the cartel rate when λ is sufficiently close to one and γ is sufficiently

---

[29]Central to a leniency program raising the cartel rate is that leniency applications are coming from cartels that have already collapsed and thus their conviction serves no desistance role. However, if a cartel that internally collapsed but was not convicted was able to reconstitute itself faster on average than a cartel that internally collapsed but was not convicted then the leniency program would promote desistance by raising the expected time until the cartel reforms and that could lower the cartel rate. In principle, the model could be enriched to allow for this effect; see footnote 9. If that effect was present, however, it may no longer be the case that it is an equilibrium for firms to apply for leniency when the cartel has collapsed. A firm would have to take into account the lower penalties from gaining leniency with the lower expected future profit from delaying the time until the cartel reforms. If the latter were greater then firms would not apply for leniency upon cartel collapse in which case the leniency program would be useless.

close to zero - numerical analysis shows that this perverse effect can occur for non-extreme parameter values. Recall that if a non-leniency case takes up one unit of CA resources then a leniency case takes up λ units. In the numerical analysis, λ ranges from 0 to 1. The penalty is the average increase in profit from collusion (relative to competition) multiplied by γ where results are reported for when γ ranges from .1 to .6, so penalties are low but are not trivial. It is also assumed that r is chosen by the CA to minimize the cartel rate.

Figures 1 and 2 reports the percentage change in the cartel rate from a leniency program when the probability of conviction, $p(\lambda L + R)$, is a linear function of the caseload and when it is a concave then convex function of the caseload, respectively.[30]

$$p(\lambda L + R) = \begin{cases} \max\{.8 - 40(\lambda L + R), 0.05\} & \text{Linear} \\ \frac{1}{1+1000(\lambda L+R)^{1.4}} & \text{Concave then convex} \end{cases}$$

When the curve lies above (below) 0, the leniency program has resulted in a rise (fall) in the cartel rate. In both figures, there is a wide array of values for λ and γ whereby the cartel rate is higher with a leniency program. In Figure 2, the cartel rate is 30% higher when a leniency program does not save on resources on a per case basis ($\lambda = 1$) and penalties are very weak ($\gamma = .1$), and a leniency program raises the cartel rate even when $\lambda = .5$ and $\gamma = .5$.

In concluding, it is not clear ex ante whether a leniency program will help or hurt the cause of anti-cartel enforcement; it very much depends on how it is implemented (is λ near one or not?) and whether there are significant penalties in place (γ is not low). To enhance the chances that a leniency program will lower the cartel rate, this analysis emphasizes the importance of appropriately setting these other enforcement instruments. It is also difficult to assess ex post the impact of a leniency program because the latent cartel rate does not move monotonically with the number of leniency applications. A leniency program can lower the cartel rate and the program itself can either be inactive (Theorem 7) or active (for example, positive volume of applications is consistent with Theorem 8), and a leniency program can raise the cartel rate even while there are leniency applications coming in (Theorem 10).[31]

5.5   Inter-Industry Variation in the Impact of a Leniency Program

Our analysis has shown how a leniency program impacts the aggregate cartel rate while taking into account its effect on non-leniency enforcement. A leniency program can lower the cartel rate by resulting in some cartels no longer forming (that is, for some industries, collusion is not incentive compatible for all market conditions) or reducing average cartel duration for those that still form. In concluding this section,

---

[30] Appendix C provides details on the numerical analysis including methods, functional forms, and values for other parameters.

[31] However, it is important to note that we are only using information from the steady-state and not the transition path. It may be possible to use information on how the number of leniency applications and discovered cartels evolve over time to assess the impact of a leniency program. An empirical analysis along those lines is conducted in Miller (2009).

let us drill down below the aggregate level to assess the potentially differential effect of a leniency program across industries.

Recall that industries are allowed to vary with respect to the parameter η where a higher value of η means a higher profit increase from cheating on the collusive arrangement. A higher value for η could be due, for example, to more firms (assuming Bertrand price competition) or a higher price elasticity to the firm demand function. When η is higher, the greater incentive to deviate means that the cartel is less stable in the sense that it will internally collapse for a wider set of market conditions (specifically, $\phi(Y, \sigma, \eta)$ is decreasing in η). This property has two implications. First, industries with sufficiently high values of η are unable to cartelize. Recall that $\widehat{\eta}$ denotes the highest value for η such that a cartel forms with positive probability. Second, when cartels are able to form ($\eta \leq \widehat{\eta}$), average cartel duration is lower when η is higher. It is straightforward to show that average duration for a cartel in a type-η industry is

$$CD(\sigma, \eta) = \frac{1}{1 - (1 - \sigma) H(\phi^*(\sigma, \eta))},$$

and that $\phi^*(\sigma, \eta)$ is non-increasing in η and is decreasing in η if $\phi^*(\sigma, \eta) \in (\underline{\pi}, \overline{\pi})$. Hence, average cartel duration is decreasing in η.

Given inter-industry variation in the presence and duration of cartels prior to a leniency program, it is natural to investigate how the impact of a leniency program varies across industries. In particular, could a leniency program make the environment less hospitable for collusion in some industries while making it more hospitable in other industries? Before presenting our results, it is useful to note that an industry type's cartel rate, $C(\sigma, \eta)$, and its average cartel duration, $CD(\sigma, \eta)$, are monotonically related:

$$C(\sigma, \eta) = \frac{\kappa(CD(\sigma, \eta) - 1)}{1 + \kappa(CD(\sigma, \eta) - 1)}.$$

Thus, in assessing how the effect of a leniency program varies across industries, we can consider its influence on either the cartel rate or cartel duration.

Figure 3 reports the average cartel duration for each industry type $\eta \leq \widehat{\eta}$ when the probability of success function is linear in caseload, while Figure 4 does the same for when it is concave then convex in caseload. First note that the introduction of a leniency program reduces $\widehat{\eta}$ and thereby shrinks the range of industry types for which a cartel forms with positive probability; for example, in Figure 3(b), $\widehat{\eta}$ falls from about 1.55 to 1.52. This reduction in $\widehat{\eta}$ was found for almost all parameterizations (see Appendix C) though there were a rare few in which a leniency program actually increased $\widehat{\eta}$. Second, for those industries that do cartelize with positive probability, a leniency program has a differential effect across industries depending on whether the industry produces relatively stable cartels (η is near $\underline{\eta}$) or unstable cartels (η is near $\widehat{\eta}$). Specifically, the effect of a leniency program on average cartel duration (or the cartel rate) is decreasing in η so that industries that produce less stable cartels tend to experience a bigger drop in cartel duration than industries with more stable cartels. This property is apparent in Figures 3 and 4 as the change in average duration is

decreasing in η, which holds as well for all other parameterizations considered (see Appendix C). Even more that, a leniency program can result in *longer* duration for the most stable cartels (change in duration is positive when η is low) while shutting down or shortening duration of the least stable cartels.

Property: A leniency program generally reduces the range of industries that are able to form cartels. The effect of a leniency program on average cartel duration is decreasing in η so that industries with less stable cartels experience a bigger decline in average cartel duration or, equivalently, a bigger decline in the percentage of industries that cartelize. This differential effect can be so significant that a leniency program can reduce average cartel duration or eliminate cartels altogether in industries with relatively unstable cartels and, at the same time, increase average cartel duration in industries with relatively stable cartels.

To understand what is driving the differential effect of a leniency program across industries, recall that it is dying cartels that use the leniency program. Once market conditions are such that collusion is no longer incentive compatible, firms stop colluding and then race to apply for leniency. Of course, only one firm will receive leniency and the remaining firms end up paying full penalties. Because the leniency program then ensures conviction when the cartel dies, expected penalties are higher with a leniency program.[32] At the same time, the flow of leniency applications can weaken non-leniency enforcement by reducing the likelihood of being prosecuted and convicted outside of the leniency program. In sum, expected penalties can be higher when discovered and prosecuted through the leniency program but can be lower when discovered and prosecuted outside of the leniency program. Which of these effects is more important depends on an industry's type. Firms in industries with relatively unstable cartels know there is a significant chance the cartel will internally collapse which will then induce a race for leniency. Thus, those cartels are especially harmed by the higher penalties coming from a leniency applicant and, therefore, they are worse off after the introduction of a leniency program. At the same time, firms in industries with relatively stable cartels are less concerned with a race for leniency because cartel collapse is unlikely and such a race only ensues in that event. The greater concern for a highly stable cartel is with non-leniency enforcement and, if that is weaker by virtue of a leniency program, they actually find expected penalties to be lower and, therefore, the environment to be more hospitable for collusion.

Complementing these numerical results, we can prove that the least stable cartels are harmed by a leniency program when non-leniency enforcement is not weakened, and the most stable cartels are benefitted by a leniency program when non-leniency enforcement is weakened. Theorem 11 shows that if an industry produces sufficiently stable cartels then the impact of a leniency program is largely determined by how it influences non-leniency enforcement. If the introduction of a leniency program

---

[32]That expected penalties in the event of cartel collapse are actually higher with a leniency program does require $\sigma^*_{NL} < \omega$ which is true (by a large margin) for all of the parameterizations considered in the numerical analysis.

strengthens non-leniency enforcement then the cartel rate (or average cartel duration) for highly stable cartels declines, while if it weakens non-leniency enforcement then the cartel rate (or average cartel duration) rises. These highly stable cartels are not concerned with the higher penalties coming from a race for leniency - because a race is unlikely for those cartels - and instead are concerned with whether they are more or less likely to be prosecuted and convicted outside of the leniency program.

Theorem 11 *Assume $\underline{\eta} = 1$ and $C_{NL}\left(\sigma_{NL}^*\right), C_L\left(\sigma_L^*, \eta\right) > 0$ then*

$$\lim_{\eta \to \underline{\eta}} \left[C_L\left(\sigma_L^*, \eta\right) - C_{NL}\left(\sigma_{NL}^*, \eta\right)\right] = \frac{\kappa\left(\sigma_{NL}^* - \sigma_L^*\right)}{\left[1 - (1-\kappa)\left(1-\sigma_L^*\right)\right]\left[1 - (1-\kappa)\left(1-\sigma_{NL}^*\right)\right]}.$$

Hence, for η close to one, $C_L\left(\sigma_L^*, \eta\right) < (>) C_{NL}\left(\sigma_{NL}^*, \eta\right)$ if and only if $\sigma_L^* > (<) \sigma_{NL}^*$.

The next result shows that, unless non-leniency enforcement is weakened, a leniency program is sure to destabilize the least stable cartels. More specifically, if $\sigma_L^* \simeq \sigma_{NL}^*$ then industries for which $\eta \in \left(\widehat{\eta}_L\left(\sigma_L^*\right), \widehat{\eta}_{NL}\left(\sigma_{NL}^*\right)\right]$ were able to collude in the absence of a leniency program but are not able to do so with a leniency program.

Theorem 12 *If $\sigma_{NL}^*, \sigma_L^* \in (0, \omega)$, $\sigma_L^* \geq \sigma_{NL}^*$, and $\widehat{\eta}_{NL}\left(\sigma_{NL}^*\right) \in \left(\underline{\eta}, \overline{\eta}\right)$ then $\widehat{\eta}_L\left(\sigma_L^*\right) < \widehat{\eta}_{NL}\left(\sigma_{NL}^*\right)$.*

In sum, the institution of a leniency program raises expected penalties and, as a result, shortens cartel duration (or prevents cartels from forming at all) in industries for which collusion is least stable, while it can actually lower expected penalties and thereby lengthen cartel duration in industries for which collusion is most stable because non-leniency enforcement is weaker. Thus, a leniency program can result in fewer cartels forming but those that form last a longer time.

6   Concluding Remarks

This paper has two main messages for scholar and practitioners. The first message is that a proper assessment of the impact of a leniency program on the frequency of cartels requires taking account of how it influences enforcement through more standard non-leniency means. Holding non-leniency enforcement fixed, we find that a leniency program generally lowers the cartel rate, which is consistent with previous theoretical research. When non-leniency enforcement is endogenized, a leniency program can either lower or raise the cartel rate. Whether there are more or less cartels depends on the extent to which leniency applications shift competition authority resources away from pursuing cases without a leniency application to cases with a leniency application. In particular, introducing a leniency program into an environment in which penalties are low and leniency cases take up comparable resources to that of non-leniency cases is predicted to result in *more* cartels. Ensuring a leniency program has the desired effect of reducing the frequency of cartels then requires proper setting of complementary instruments. Specifically, penalties should not be weak and a

29

procedure should be devised to handle leniency applications in an expedited manner in order to reduce the amount of resources involved.

The second message is that the success of a leniency program should not be measured by the volume of applications. While activity in a leniency program is not to be ignored, more applications do not translate into fewer cartels. Though this message has been said before and by others, the perils of not heeding it come through rather strikingly in the model of this paper. Furthermore, it is a message worth repeating because various practitioners continue to equate a high volume of leniency applications with a successful program, when instead a successful program should be measured by fewer and less damaging cartels. Let me provide two recent examples. The first comes from two private antitrust lawyers who were commenting on a recent U.S. GAO Report that evaluated the 2004 Antitrust Criminal Penalty Enforcement and Reform Act (ACPERA):[33]

> While the DOJ is unsurprisingly pleased with the marked increase in the amount of criminal penalties obtained and jail sentences imposed on offenders, the total number of leniency applications is up only a modest 4 percent in the six years since ACPERA's enactment compared to the previous six-year period.

Contrary to the concern they posed, an effective leniency program should eventually realize a dwindling number of leniency applications because there are fewer cartels, but then we must distinguish that explanation for fewer applications from the alternative one that cartels are devising ways in which to counteract the incentives to apply for leniency. A second example is from Germany's competition authority, the Bundeskartellamt:[34]

> The first version of the Leniency Programme was already a success. This can be seen by the number of leniency applications filed: Between 2000 and 2005 a total of 122 leniency applications were filed. Under the new Leniency Programme, a further 112 were filed in only four years (2006-2009). Thus, more than 230 leniency applications have already contributed to successfully uncover, end and punish cartel agreements.

Our point is not to suggest that leniency programs are ineffective. One can identify many cases in which the leniency program was instrumental in discovering a cartel and essential for effectively prosecuting a case. Rather, the point is that we should not lose sight of the ultimate goal of leniency programs, which is to reduce the presence of collusion in an economy. Only by keeping our sights on that goal can we hope to devise methods for surmounting the considerable challenge of measuring the impact of anti-cartel programs on the unobserved cartel rate. Until that occurs, we will not really know whether we are winning or losing the fight against cartels.

---

[33]Niall Lynch and Kathleen Fox, "How ACPERA Has Affected Criminal Cartel Enforcement," *Law 360*, 11 August 2011.

[34]Bundeskartellamt, "Effective Cartel Prosecution: Benefits for the Economy and Consumers," 2010, p. 19.

7 Appendix A: Proofs

Proof of Lemma 1. ψ can be presented as:

$$(14)$$

$$
\begin{aligned}
\psi\left(Y\right) &= \int_{\underline{\pi}}^{\phi(Y)}\left[(1-\delta)\,\pi+\delta Y-\delta\sigma\left(Y-W\right)-(1-\delta)\,\sigma\gamma\left(Y-\alpha\mu\right)\right]h\left(\pi\right)d\pi \\
&+\int_{\phi(Y)}^{\overline{\pi}}\left[(1-\delta)\,\alpha\pi+\delta W-(1-\delta)\,\zeta\left(\theta,\sigma\right)\gamma\left(Y-\alpha\mu\right)\right]h\left(\pi\right)d\pi
\end{aligned}
$$

where

$$
\phi\left(Y,\sigma,\eta\right)=\frac{\delta\left(1-\sigma\right)\left(1-\kappa\right)\left(Y-\alpha\mu\right)-\left[1-\delta\left(1-\kappa\right)\right]\left[\sigma-\min\left\{\sigma,\theta\right\}\right]\gamma\left(Y-\alpha\mu\right)}{\left(\eta-1\right)\left[1-\delta\left(1-\kappa\right)\right]}
$$

$$(15)$$

and

$$
\zeta\left(\theta,\sigma\right)=\left\{\begin{array}{ll}\sigma & \text{if }\sigma\leq\theta\\\omega & \text{if }\sigma>\theta\end{array}\right. \tag{16}
$$

Evaluate (14) at $Y=\mu$ and set $W=\mu$ which gives an upper bound on $\psi\left(\mu\right)$ since $W\leq Y$ and ψ is increasing in W:

$$
\begin{aligned}
\psi\left(\mu\right) &\leq \int_{\underline{\pi}}^{\phi(\mu)}\left[(1-\delta)\,\pi+\delta\mu-(1-\delta)\,\sigma\gamma\mu\left(1-\alpha\right)\right]h\left(\pi\right)d\pi \\
&+\int_{\phi(\mu)}^{\overline{\pi}}\left[(1-\delta)\,\alpha\pi+\delta\mu-(1-\delta)\,\zeta\left(\theta,\sigma\right)\gamma\mu\left(1-\alpha\right)\right]h\left(\pi\right)d\pi
\end{aligned}
$$

$$
\begin{aligned}
\psi\left(\mu\right) &\leq \int_{\underline{\pi}}^{\phi(\mu)}\left[(1-\delta)\,\pi+\delta\mu\right]h\left(\pi\right)d\pi-\int_{\underline{\pi}}^{\phi(\mu)}\left(1-\delta\right)\sigma\gamma\mu\left(1-\alpha\right)h\left(\pi\right)d\pi \\
&+\int_{\phi(\mu)}^{\overline{\pi}}\left[(1-\delta)\,\alpha\pi+\delta\mu-(1-\delta)\,\zeta\left(\theta,\sigma\right)\gamma\mu\left(1-\alpha\right)\right]h\left(\pi\right)d\pi
\end{aligned}
$$

$$
\begin{aligned}
\psi\left(\mu\right) &\leq \delta\mu+\int_{\underline{\pi}}^{\phi(\mu)}\left(1-\delta\right)\pi h\left(\pi\right)d\pi+\int_{\phi(\mu)}^{\overline{\pi}}\left(1-\delta\right)\alpha\pi h\left(\pi\right)d\pi \\
&-\int_{\underline{\pi}}^{\phi(\mu)}\left(1-\delta\right)\sigma\gamma\mu\left(1-\alpha\right)h\left(\pi\right)d\pi-\int_{\phi(\mu)}^{\overline{\pi}}\left(1-\delta\right)\zeta\left(\theta,\sigma\right)\gamma\mu\left(1-\alpha\right)h\left(\pi\right)d\pi
\end{aligned}
$$

$$
\begin{aligned}
\psi\left(\mu\right) &\leq \delta\mu+\int_{\underline{\pi}}^{\overline{\pi}}\left(1-\delta\right)\pi h\left(\pi\right)d\pi-\int_{\phi(\mu)}^{\overline{\pi}}\left(1-\delta\right)\left(1-\alpha\right)\pi h\left(\pi\right)d\pi \\
&-\int_{\underline{\pi}}^{\phi(\mu)}\left(1-\delta\right)\sigma\gamma\mu\left(1-\alpha\right)h\left(\pi\right)d\pi-\int_{\phi(\mu)}^{\overline{\pi}}\left(1-\delta\right)\zeta\left(\theta,\sigma\right)\gamma\mu\left(1-\alpha\right)h\left(\pi\right)d\pi
\end{aligned}
$$

$$\psi\left(\mu\right) \leq \mu - \int_{\phi\left(\mu\right)}^{\overline{\pi}} \left(1-\delta\right)\left(1-\alpha\right)\pi h\left(\pi\right) d\pi - \int_{\underline{\pi}}^{\phi\left(\mu\right)} \left(1-\delta\right)\sigma\gamma\mu\left(1-\alpha\right)h\left(\pi\right)d\pi$$
$$- \int_{\phi\left(\mu\right)}^{\overline{\pi}} \left(1-\delta\right)\zeta\left(\theta,\sigma\right)\gamma\mu\left(1-\alpha\right)h\left(\pi\right)d\pi.$$

Therefore, $\psi\left(\mu\right) \leq \mu$. Given that $\psi\left(\alpha\mu\right) = \alpha\mu$, it follows that ψ maps $[\alpha\mu, \mu]$ into itself as long as $\psi'\left(Y\right) \geq 0$. We'll now show that property holds.

To prove $\psi'\left(Y\right) \geq 0$ when $\gamma \simeq 0$, begin with when $\phi\left(Y\right) < \underline{\pi}$ so that (14) takes the form:

$$\psi\left(Y\right) = \int_{\underline{\pi}}^{\overline{\pi}} \left[\left(1-\delta\right)\alpha\pi + \delta W - \left(1-\delta\right)\zeta\left(\theta,\sigma\right)\gamma\left(Y-\alpha\mu\right)\right]h\left(\pi\right)d\pi.$$

Thus,

$$\begin{aligned}
\psi'\left(Y\right) &= \int_{\underline{\pi}}^{\overline{\pi}} \left[\delta\frac{\partial W}{\partial Y} - \left(1-\delta\right)\zeta\left(\theta,\sigma\right)\gamma\right]h\left(\pi\right)d\pi \\
&= \frac{\delta\kappa}{1-\delta\left(1-\kappa\right)} - \left(1-\delta\right)\zeta\left(\theta,\sigma\right)\gamma
\end{aligned}$$

and $\psi'\left(Y\right) \in \left(0,1\right)$ when $\gamma \simeq 0$. If $\overline{\pi} < \phi\left(Y\right)$ then (14) takes the form:

$$\psi\left(Y\right) = \int_{\underline{\pi}}^{\overline{\pi}} \left[\left(1-\delta\right)\pi + \delta Y - \delta\sigma\left(Y-W\right)\right]h\left(\pi\right)d\pi - \left(1-\delta\right)\sigma\gamma\left(Y-\alpha\mu\right),$$

and

$$\psi'\left(Y\right) = \delta\left[1 - \sigma\left(1-\frac{\partial W}{\partial Y}\right) - \left(1-\delta\right)\sigma\gamma\right] = \delta\left[\left(1-\sigma\right)\left(1-\frac{\partial W}{\partial Y}\right) + \frac{\partial W}{\partial Y}\right] - \left(1-\delta\right)\sigma\gamma.$$

Given that $\partial W/\partial Y = \kappa/\left(1-\delta\left(1-\kappa\right)\right) \in \left(0,1\right)$, if $\gamma \simeq 0$ then $\psi'\left(Y\right) \in \left(0,1\right)$. Finally, if $\phi\left(Y\right) \in \left(\underline{\pi}, \overline{\pi}\right)$ then differentiating (14) yields:

$$\begin{aligned}
\psi'\left(Y\right) &= \left[\left(1-\delta\right)\phi\left(Y\right) + \delta Y - \delta\sigma\left(Y-W\right) - \left(1-\delta\right)\sigma\gamma\left(Y-\alpha\mu\right)\right]h\left(\phi\left(Y\right)\right)\phi'\left(Y\right) \\
&\quad + \int_{\underline{\pi}}^{\phi\left(Y\right)} \left\{\delta\left[1 - \sigma\left(1-\frac{\partial W}{\partial Y}\right)\right] - \left(1-\delta\right)\sigma\gamma\right\}h\left(\pi\right)d\pi \\
&\quad - \left[\left(1-\delta\right)\alpha\phi\left(Y\right) + \delta W - \left(1-\delta\right)\zeta\left(\theta,\sigma\right)\gamma\left(Y-\alpha\mu\right)\right]h\left(\phi\left(Y\right)\right)\phi'\left(Y\right) \\
&\quad + \int_{\phi\left(Y\right)}^{\overline{\pi}} \left[\delta\frac{\partial W}{\partial Y} - \left(1-\delta\right)\zeta\left(\theta,\sigma\right)\gamma\right]h\left(\pi\right)d\pi
\end{aligned}$$

$$\begin{aligned}
\psi'\left(Y\right) &= \left[\left(1-\delta\right)\left(1-\alpha\right)\phi\left(Y\right) + \delta\left(1-\sigma\right)\left(Y-W\right)\right]h\left(\phi\left(Y\right)\right)\phi'\left(Y\right) \qquad\qquad (17) \\
&\quad + \int_{\underline{\pi}}^{\phi\left(Y\right)} \delta\left(1-\sigma\right)\left(1-\frac{\partial W}{\partial Y}\right)h\left(\pi\right)d\pi + \delta\frac{\partial W}{\partial Y} \\
&\quad + \gamma\left(1-\delta\right)\left[\left(\zeta\left(\theta,\sigma\right)-\sigma\right)\left(Y-\alpha\mu\right)h\left(\phi\left(Y\right)\right)\phi'\left(Y\right) - \sigma H\left(\phi\left(Y\right)\right) + \zeta\left(\theta,\sigma\right)\left(1-H\left(\phi\left(Y\right)\right)\right)\right]
\end{aligned}$$

32

In evaluating the sign of $\psi'(Y)$, note that if $\gamma \simeq 0$ then

$$\phi'(Y) = \frac{\delta(1-\sigma)(1-\kappa)}{(1-\delta(1-\kappa))(\eta-1)} - \frac{[1-\delta(1-\kappa)][\sigma-\min\{\sigma,\theta\}]\gamma}{\eta-1} > 0.$$

(17) sums up four terms. The first term is positive. Next note that $\partial W/\partial Y \in (0,1)$ implies the second and third terms are positive. If $\gamma \simeq 0$ then the fourth term is small relative to the first three terms which implies $\psi'(Y) > 0$. ∎

Proof of Lemma 2. First, let us consider the impact on $Y^*(\sigma,\eta)$ of changing σ. Initially, let us suppose that $\phi^*(\sigma,\eta) \in (\underline{\pi},\overline{\pi})$. Referring to (14)-(16) and given that $\omega > \theta$, there is a discontinuous decrease in $\psi(Y)$ at $\sigma = \theta$ when σ is increased as the penalty jumps from $\theta\gamma(Y-\alpha\mu)$ to $\omega\gamma(Y-\alpha\mu)$. Thus, at $\sigma = \theta$, $\psi(Y)$ is decreasing in σ. Next consider the response of $\psi(Y)$ to σ when $\sigma \neq \theta$ and thus is differentiable:

$$\frac{\partial\psi(Y)}{\partial\sigma} = [(1-\delta)\phi + \delta Y - \delta\sigma(Y-W) - (1-\delta)\sigma\gamma(Y-\alpha\mu)]h(\phi)\frac{\partial\phi}{\partial\sigma}$$

$$- \int_{\underline{\pi}}^{\phi(Y)}[\delta(Y-W) + (1-\delta)\gamma(Y-\alpha\mu)]h(\pi)\,d\pi$$

$$- [(1-\delta)\alpha\phi + \delta W - (1-\delta)\zeta(\theta,\sigma)\gamma(Y-\alpha\mu)]h(\phi)\frac{\partial\phi}{\partial\sigma}$$

$$- \int_{\phi(Y)}^{\overline{\pi}}(1-\delta)\frac{\partial\zeta(\theta,\sigma)}{\partial\sigma}\gamma(Y-\alpha\mu)h(\pi)\,d\pi$$

$$\frac{\partial\psi(Y)}{\partial\sigma} = [(1-\delta)(1-\alpha)\phi + \delta(1-\sigma)(Y-W)]h(\phi)\frac{\partial\phi}{\partial\sigma} \qquad (18)$$

$$- \int_{\underline{\pi}}^{\phi(Y)}[\delta(Y-W) + (1-\delta)\gamma(Y-\alpha\mu)]h(\pi)\,d\pi$$

$$- \int_{\phi(Y)}^{\overline{\pi}}(1-\delta)\frac{\partial\zeta(\theta,\sigma)}{\partial\sigma}\gamma(Y-\alpha\mu)h(\pi)\,d\pi$$

$$+ (1-\delta)(\zeta(\theta,\sigma)-\sigma)\gamma(Y-\alpha\mu)h(\phi)\frac{\partial\phi}{\partial\sigma}$$

where
$$\frac{\partial\zeta(\theta,\sigma)}{\partial\sigma} = \begin{cases} 1 & \text{if } \sigma < \theta \\ 0 & \text{if } \sigma > \theta \end{cases}$$

In signing these terms, recall that we are focusing on the case of $\phi^*(\sigma,\eta) \in (\underline{\pi},\overline{\pi})$ which implies $Y^*(\sigma,\eta) > \alpha\mu$. Given that

$$\frac{\partial\phi}{\partial\sigma} = -\left(\frac{\delta(1-\kappa)(Y-\alpha\mu)}{(1-\delta(1-\kappa))(\eta-1)}\right) - \frac{\mathbf{1}_{\sigma>\theta}\gamma(Y-\alpha\mu)}{\eta-1} < 0,$$

where $\mathbf{1}_{\sigma>\theta} = 1$ if $\sigma > \theta$ and 0 otherwise, then the first term in (18) is negative. The second term is negative and the third term is non-positive. If $\gamma \simeq 0$ then the fourth term is small relative to the first two terms from which we can conclude $\partial\psi(Y)/\partial\sigma <$

33

0. Now consider increasing σ from σ' to σ''. Given that $\psi(Y)$ is decreasing in σ then $\psi(Y)$ shifts down. Since $\psi(Y, \sigma') \leq Y$ as $Y \geq Y^*(\sigma', \eta)$ (recalling that Y^* is the maximal fixed point) then $\psi(Y, \sigma'') < Y$ for all $Y \geq Y^*(\sigma', \eta)$ which implies $Y^*(\sigma'', \eta) < Y^*(\sigma', \eta)$. Thus, if $\phi^*(\sigma, \eta) \in (\underline{\pi}, \overline{\pi})$, which implies $Y^*(\sigma, \eta) > \alpha\mu$, then $Y^*(\sigma, \eta)$ is decreasing in σ.

Next suppose $\phi^*(\sigma, \eta) = \overline{\pi}$ in which case

$$\psi(Y) = \int_{\underline{\pi}}^{\overline{\pi}} [(1-\delta)\pi + \delta Y - \delta\sigma(Y - W) - (1-\delta)\sigma\gamma(Y - \alpha\mu)] h(\pi) d\pi$$

and

$$\frac{\partial\psi(Y)}{\partial\sigma} = -\int_{\underline{\pi}}^{\overline{\pi}} [\delta(Y - W) + (1-\delta)\gamma(Y - \alpha\mu)] h(\pi) d\pi < 0.$$

By the previous argument, it follows that $Y^*(\sigma, \eta)$ is decreasing in σ. Note that if $\phi^*(\sigma, \eta) = \overline{\pi}$ then $Y^*(\sigma, \eta) > \alpha\mu$.

Finally, suppose $\phi^*(\sigma, \eta) = \underline{\pi}$. Given that $Y^*(\sigma, \eta) = \alpha\mu$ then $Y^*(\sigma, \eta)$ is independent of σ. Summing up, it has been shown that $Y^*(\sigma, \eta)$ is non-increasing in σ and, when $Y^*(\sigma, \eta) > \alpha\mu$, $Y^*(\sigma, \eta)$ is decreasing in σ.

Now consider the impact on $Y^*(\sigma, \eta)$ of changing η. Note that η only operates through ϕ since, in equilibrium, a firm never cheats. Hence, if $\phi^*(\sigma, \eta) \in \{\underline{\pi}, \overline{\pi}\}$ then $Y^*(\sigma, \eta)$ is independent of η. Let us then suppose $\phi^*(\sigma, \eta) \in (\underline{\pi}, \overline{\pi})$ in which case

$$
\begin{aligned}
\psi(Y) &= \int_{\underline{\pi}}^{\phi(Y)} [(1-\delta)\pi + \delta Y - \delta\sigma(Y - W) - (1-\delta)\sigma\gamma(Y - \alpha\mu)] h(\pi) d\pi \\
&+ \int_{\phi(Y)}^{\overline{\pi}} [(1-\delta)\alpha\pi + \delta W - (1-\delta)\zeta(\theta, \sigma)\gamma(Y - \alpha\mu)] h(\pi) d\pi
\end{aligned}
$$

at $Y = Y^*(\sigma, \eta)$. We then have

$$
\begin{aligned}
\frac{\partial\psi}{\partial\eta} &= \{[(1-\delta)\phi + \delta Y - \delta\sigma(Y - W) - (1-\delta)\sigma\gamma(Y - \alpha\mu)] \\
&\quad - (1-\delta)\alpha\phi - \delta W + (1-\delta)\zeta(\theta, \sigma)\gamma(Y - \alpha\mu)\} h(\phi)\left(\frac{\partial\phi}{\partial\eta}\right) \\
&= \{[(1-\delta)(1-\alpha)\phi + \delta(1-\sigma)(Y - W) + (1-\delta)(\zeta(\theta, \sigma) - \sigma)\gamma(Y - \alpha\mu)]\} h(\phi)\left(\frac{\partial\phi}{\partial\eta}\right)
\end{aligned}
$$

If $\gamma \simeq 0$ then the expression in $\{\}$ is positive in which case

$$sign\left\{\frac{\partial\psi}{\partial\eta}\right\} = sign\left\{\frac{\partial\phi}{\partial\eta}\right\}.$$

Given that

$$\frac{\partial\phi}{\partial\eta} = -\left(\frac{\delta(1-\sigma)(1-\kappa)(Y - \alpha\mu) - [1 - \delta(1-\kappa)][\sigma - \min\{\sigma, \theta\}]\gamma(Y - \alpha\mu)}{(\eta - 1)^2[1 - \delta(1-\kappa)]}\right) = -\frac{\phi}{\eta - 1} < 0$$

then $\partial\psi/\partial\eta < 0$. Hence, raising η lowers $\psi(Y, \sigma, \eta)$ and thus lowers $Y^*(\sigma, \eta)$. We have then shown that $Y^*(\sigma, \eta)$ is non-increasing in η and, when $\phi^*(\sigma, \eta) \in (\underline{\pi}, \overline{\pi})$, $Y^*(\sigma, \eta)$ is decreasing in η.

Turning to comparative statics for $\phi^*(\sigma, \eta)$, recall that

$$\phi^*(\sigma, \eta) \equiv \max\{\min\{\phi(Y^*(\sigma, \eta), \sigma, \eta), \overline{\pi}\}, \underline{\pi}\}$$

where the expression for $\phi(Y, \sigma, \eta)$ is in (15). If $\phi^*(\sigma, \eta) \in \{\underline{\pi}, \overline{\pi}\}$ then it is independent of σ and η. Thus, from hereon assume $\phi^*(\sigma, \eta) \in (\underline{\pi}, \overline{\pi})$. In that case,

$$\frac{\partial\phi(Y^*(\sigma, \eta), \sigma, \eta)}{\partial Y} = \frac{\delta(1-\sigma)(1-\kappa) - [1-\delta(1-\kappa)][\sigma - \min\{\sigma, \theta\}]\gamma}{(\eta - 1)[1 - \delta(1-\kappa)]}$$

and

$$\frac{\partial\phi(Y^*(\sigma, \eta), \sigma, \eta)}{\partial \sigma} = \begin{cases} -\left(\frac{\delta(1-\kappa)(Y^*(\sigma,\eta)-\alpha\mu)+[1-\delta(1-\kappa)]\gamma(Y^*(\sigma,\eta)-\alpha\mu)}{(\eta-1)[1-\delta(1-\kappa)]}\right) & \text{if } \sigma > \theta \\[2ex] -\frac{\delta(1-\kappa)(Y^*(\sigma,\eta)-\alpha\mu)}{(\eta-1)[1-\delta(1-\kappa)]} & \text{if } \sigma < \theta \end{cases}.$$

$\phi(Y, \sigma, \eta)$ is then decreasing in σ and, when $\gamma \simeq 0$, increasing in Y. Given that it has already been shown that $Y^*(\sigma, \eta)$ is decreasing in σ, it follows that $\phi^*(\sigma, \eta)$ is decreasing in σ.

Finally, consider the comparative static of $\phi^*(\sigma, \eta)$ with respect to η. If $\phi^*(\sigma, \eta) \in (\underline{\pi}, \overline{\pi})$ then re-arranging (15) yields

$$\phi(Y, \sigma, \eta) = \left(\frac{\delta(1-\sigma)(1-\kappa) - [1-\delta(1-\kappa)][\sigma - \min\{\sigma, \theta\}]\gamma}{1 - \delta(1-\kappa)}\right)\left(\frac{Y - \alpha\mu}{\eta - 1}\right).$$

Consider $\eta'' > \eta'$,

$$\phi^*(\sigma, \eta'') - \phi^*(\sigma, \eta') = \left(\frac{\delta(1-\sigma)(1-\kappa) - [1-\delta(1-\kappa)][\sigma - \min\{\sigma, \theta\}]\gamma}{1 - \delta(1-\kappa)}\right) \times$$
$$\left[\left(\frac{Y^*(\sigma, \eta'') - \alpha\mu}{\eta'' - 1}\right) - \left(\frac{Y^*(\sigma, \eta') - \alpha\mu}{\eta'' - 1}\right)\right].$$

If $\gamma \simeq 0$ then

$$\frac{\delta(1-\sigma)(1-\kappa) - [1-\delta(1-\kappa)][\sigma - \min\{\sigma, \theta\}]\gamma}{1 - \delta(1-\kappa)} > 0$$

and, therefore,

$$sign\{\phi^*(\sigma, \eta'') - \phi^*(\sigma, \eta')\} = sign\left\{\left(\frac{Y^*(\sigma, \eta'') - \alpha\mu}{\eta'' - 1}\right) - \left(\frac{Y^*(\sigma, \eta') - \alpha\mu}{\eta' - 1}\right)\right\}.$$

This expression is negative because $\eta'' - 1 > \eta' - 1 > 0$ and, given that $Y^*(\sigma, \eta)$ is decreasing in η, $Y^*(\sigma, \eta') - \alpha\mu > Y^*(\sigma, \eta'') - \alpha\mu > 0$. This proves $\phi^*(\sigma, \eta)$ is decreasing in η. ∎

Proof of Lemma 3. [The method of proof is the same as that used in proving Theorem 4 in Harrington and Chang (2009).] Recall that $\widehat{\eta}(\sigma)$ satisfies the property,

$$Y^*(\sigma, \eta) \begin{cases} > \alpha\mu & \text{if } \eta \in \left[\underline{\eta}, \widehat{\eta}(\sigma)\right] \\ = \alpha\mu & \text{if } \eta > \widehat{\eta}(\sigma) \end{cases}$$

Suppose $\widehat{\eta}(\sigma) \in \left(\underline{\eta}, \overline{\eta}\right)$. By the continuity of $\psi(Y, \sigma, \eta)$ with respect to Y and η and that $\psi(Y, \sigma, \eta)$ is non-increasing in η (see the proof of Lemma 2), it follows that

$$\psi(Y, \widehat{\eta}(\sigma)) \leq Y, \text{ for all } Y, \text{ and } \psi(Y^*(\sigma, \widehat{\eta}(\sigma)), \widehat{\eta}(\sigma)) = Y^*(\sigma, \widehat{\eta}(\sigma)).$$

With this property, let us now argue that $\widehat{\eta}$ is decreasing in σ. Consider raising σ from σ' to σ''. Since $\psi(Y, \eta)$ is decreasing in σ for $Y > \alpha\mu$ (see the proof of Lemma 2) then

$$\psi\left(Y, \widehat{\eta}\left(\sigma'\right), \sigma''\right) < Y, \text{ for all } Y > \alpha\mu.$$

Given that $\psi(Y, \eta)$ is decreasing in η for $Y > \alpha\mu$ then $\widehat{\eta}(\sigma'') < \widehat{\eta}(\sigma')$. ∎

Proof of Theorem 4. [The method of proof is the same as that used in proving in Harrington and Chang (2009).] Recall that the cartel rate for a type-η industry is

$$C(\sigma, \eta) = \frac{\kappa(1 - \sigma) H(\phi^*(\sigma, \eta))}{1 - (1 - \kappa)(1 - \sigma) H(\phi^*(\sigma, \eta))}.$$

Suppose σ is raised from σ' to σ''. The change in the type-η industry cartel rate is

$$\frac{\kappa(1 - \sigma'') H(\phi^*(\sigma'', \eta))}{1 - (1 - \kappa)(1 - \sigma'') H(\phi^*(\sigma'', \eta))} - \frac{\kappa(1 - \sigma') H(\phi^*(\sigma', \eta))}{1 - (1 - \kappa)(1 - \sigma') H(\phi^*(\sigma', \eta))}.$$

The sign of that expression is the same as

$$\begin{aligned} &sign\left\{\kappa\left(1 - \sigma''\right) H\left(\phi^*\left(\sigma'', \eta\right)\right)\left[1 - (1 - \kappa)\left(1 - \sigma'\right) H\left(\phi^*\left(\sigma', \eta\right)\right)\right] - \right. \\ &\left. \kappa\left(1 - \sigma'\right) H\left(\phi^*\left(\sigma', \eta\right)\right)\left[1 - (1 - \kappa)\left(1 - \sigma''\right) H\left(\phi^*\left(\sigma'', \eta\right)\right)\right]\right\} \\ = \ &sign\left\{\left(1 - \sigma''\right) H\left(\phi^*\left(\sigma'', \eta\right)\right) - \left(1 - \sigma'\right) H\left(\phi^*\left(\sigma', \eta\right)\right)\right\} < 0. \end{aligned}$$

This expression is negative because $\sigma'' > \sigma'$ implies $1 - \sigma'' < 1 - \sigma'$, and $H(\phi^*(\sigma'', \eta)) < H(\phi^*(\sigma', \eta))$ because $\phi^*(\sigma'', \eta) < \phi^*(\sigma', \eta)$ (by Lemma 2).

Next consider the aggregate cartel rate,

$$\begin{aligned} C(\sigma) &= \int_{\underline{\eta}}^{\overline{\eta}} \left[\frac{\kappa(1 - \sigma) H(\phi^*(\sigma, \eta))}{1 - (1 - \kappa)(1 - \sigma) H(\phi^*(\sigma, \eta))}\right] g(\eta) \, d\eta \\ &= \int_{\underline{\eta}}^{\widehat{\eta}(\sigma)} \left[\frac{\kappa(1 - \sigma) H(\phi^*(\sigma, \eta))}{1 - (1 - \kappa)(1 - \sigma) H(\phi^*(\sigma, \eta))}\right] g(\eta) \, d\eta \end{aligned}$$

It was just shown that the integrand is decreasing in σ and, since $\widehat{\eta}(\sigma)$ is decreasing in σ by Lemma 3, this expression is decreasing in σ. ∎

Proof of Theorem 5. When $\theta = 1$ then

$$\Psi(\sigma) = qrp\left(qr \int_{\underline{\eta}}^{\overline{\eta}} C(\sigma, \eta) g(\eta) d\eta\right), \tag{19}$$

and when $\theta = 0$ then

$$\begin{aligned}\Psi(\sigma) &= qrp\left(\lambda \int_{\underline{\eta}}^{\overline{\eta}} (1 - H(\phi^*(\sigma, \eta))) C(\sigma, \eta) g(\eta) d\eta \right. \tag{20} \\ &\quad \left. + qr \int_{\underline{\eta}}^{\overline{\eta}} H(\phi^*(\sigma, \eta)) C(\sigma, \eta) g(\eta) d\eta\right).\end{aligned}$$

To show that a fixed point exists for (19) and for (20), the proof strategy has two steps: 1) show that, for any value of σ, the integrand in these equations is continuous in σ except for a countable set of values of η; and 2) show that it follows from step 1 that Ψ is continuous. The proof will focus exclusively on proving that (20) has a fixed point as the method of proof is immediately applicable to the case of (19).[34]

Considering the integrand in (20), a discontinuity in

$$H(\phi^*(\sigma, \eta)) C(\sigma, \eta) g(\eta) = H(\phi^*(\sigma, \eta)) \left(\frac{\kappa (1 - \sigma) H(\phi^*(\sigma, \eta))}{1 - (1 - \kappa)(1 - \sigma) H(\phi^*(\sigma, \eta))}\right) g(\eta)$$

with respect to σ (or η) comes from $\phi^*(\sigma, \eta)$ being discontinuous, which comes from $Y^*(\sigma, \eta)$ being discontinuous. Let $\Delta(\sigma') \subseteq [\underline{\eta}, \overline{\eta}]$ be the set of η for which $Y^*(\sigma, \eta)$ is discontinuous at $\sigma = \sigma'$. We will show that $\Delta(\sigma)$ is countable.

Suppose $Y^*(\sigma, \eta)$ is discontinuous in σ at $(\sigma, \eta) = (\sigma', \eta')$. Given $\psi(Y, \sigma, \eta)$ is continuous and $Y^*(\sigma, \eta)$ is the maximal fixed point to $\psi(Y, \sigma, \eta)$ then

$$\psi(Y, \sigma', \eta') < Y, \forall Y \in (Y^*(\sigma', \eta'), \mu]. \tag{21}$$

If, in addition, $\exists \xi > 0$ such that

$$\psi(Y, \sigma', \eta') > Y, \forall Y \in [Y^*(\sigma', \eta') - \xi, Y^*(\sigma', \eta'))$$

then, by the continuity of $\psi(Y, \sigma, \eta)$ in σ, $Y^*(\sigma, \eta)$ is continuous at $(\sigma, \eta) = (\sigma', \eta')$, contrary to our supposition. Hence, it must be the case that $\exists \xi > 0$ such that

$$\psi(Y, \sigma', \eta') \leq Y, \forall Y \in [Y^*(\sigma', \eta') - \xi, Y^*(\sigma', \eta')]. \tag{22}$$

Figures 5a-5c cover the possible cases in which Y^* is discontinuous.

Given that $\psi(Y, \sigma, \eta)$ is continuous and decreasing in η (see the proof of Lemma 2) then (21) and (22) imply

$$\psi(Y, \sigma', \eta') < Y, \forall Y \in [Y^*(\sigma', \eta') - \xi, \mu], \ \forall \eta > \eta'. \tag{23}$$

---

[34]When $\theta = 1$, existence of a fixed point can also be established by showing that $\Psi(\sigma)$ is non-decreasing in σ and appealing to Tarski's Fixed Point Theorem. However, when $\theta < 1$, it is generally not true that $\Psi(\sigma)$ is non-decreasing in $\sigma \ \forall \sigma$.

It follows from (23) that, $\forall \eta > \eta'$, all fixed points to ψ are bounded above by $Y^*(\sigma', \eta') - \xi$:

$$Y^*(\sigma', \eta) < Y^*(\sigma', \eta') - \xi, \ \forall \eta > \eta'.$$

Next define:

$$\varepsilon(\sigma', \eta') \equiv Y^*(\sigma', \eta') - \lim_{\eta \downarrow \eta'} Y^*(\sigma', \eta)$$

where $\varepsilon(\sigma', \eta')$ measures the size of the discontinuity in $Y^*(\sigma', \eta)$ with respect to η at $\eta = \eta'$; see Figure 6.

For each $\eta \in \Delta(\sigma')$, there has then been associated an interval of length $\varepsilon(\sigma', \eta)$. Note that these intervals have a null intersection because $Y^*(\sigma, \eta)$ is non-increasing in η. Hence,

$$\sum_{\eta \in \Delta(\sigma')} \varepsilon(\sigma', \eta) \leq (1 - \alpha) \mu.$$

Given that a sum can only be finite if the number of elements which are positive is countable, it follows that $\Delta(\sigma')$ is countable. Hence, the set of values for η for which $Y^*(\sigma', \eta)$ is discontinuous in σ at $\sigma = \sigma'$ is countable. This completes the first step.

By Jeffrey (1925), given that $H(\phi^*(\sigma, \eta)) C(\sigma, \eta) g(\eta)$ and $(1 - H(\phi^*(\sigma, \eta))) C(\sigma, \eta) g(\eta)$ are bounded in (σ, η) on $[0, 1] \times [\underline{\eta}, \overline{\eta}]$ and are continuous at $\sigma = \sigma'$ for all $\eta \in [\underline{\eta}, \overline{\eta}]$ except for a countable set then

$$\int_{\underline{\eta}}^{\overline{\eta}} H(\phi^*(\sigma, \eta)) C(\sigma, \eta) g(\eta) \, d\eta$$

and

$$\int_{\underline{\eta}}^{\overline{\eta}} (1 - H(\phi^*(\sigma, \eta))) C(\sigma, \eta) g(\eta) \, d\eta$$

are continuous at $\sigma = \sigma'$. Given that p is a continuous function, it follows that

$$p\left(\lambda \int_{\underline{\eta}}^{\overline{\eta}} (1 - H(\phi^*(\sigma, \eta))) C(\sigma, \eta) g(\eta) \, d\eta + qr \int_{\underline{\eta}}^{\overline{\eta}} H(\phi^*(\sigma, \eta)) C(\sigma, \eta) g(\eta) \, d\eta \right)$$

is continuous in σ. Hence, Ψ in (20) is continuous in σ and maps $[0, 1]$ into itself; therefore, a fixed point exists. The same method of proof can be used to show that a fixed point to (19) exists. ∎

Proof of Theorem 6. The proof has three steps. First, holding Y fixed, the threshold for stable collusion is shown to be lower with a leniency program: $\phi_{NL}(Y, \eta) > \phi_\theta(Y, \eta)$. When $\sigma > \theta$, which holds by supposition, the deviator has lower penalties by applying for leniency and this tightens the ICC and thus raises the threshold. Second, given $\phi_{NL}(Y, \eta) > \phi_\theta(Y, \eta)$ and the supposition that $\omega > \sigma$, it is shown that $\psi_{NL}(Y, \sigma, \eta) \geq \psi_\theta(Y, \sigma, \eta)$. That the collusive value function is weakly lower with a leniency program is due to two effects: i) $\phi_{NL}(Y, \eta) > \phi_\theta(Y, \eta)$ results in weakly shorter cartel duration with a leniency program; and ii) when there is a leniency program, expected penalties upon cartel collapse are

$\omega\gamma\left(Y-\alpha\mu\right)$ rather than $\sigma\gamma\left(Y-\alpha\mu\right)$, and the former are higher when $\omega>\sigma$. Third, $\psi_{NL}\left(Y,\sigma,\eta\right)>\psi_{\theta}\left(Y,\sigma,\eta\right)$ implies a weakly lower fixed point with a leniency program - $Y_{NL}^{*}\left(\sigma,\eta\right)\geq Y_{\theta}^{*}\left(\sigma,\eta\right)$ - and, therefore, a weakly lower equilibrium threshold: $\phi_{NL}^{*}\left(\sigma,\eta\right)\geq\phi_{\theta}^{*}\left(\sigma,\eta\right)$. This proves the cartel rate is no higher with a leniency program. If, in addition, there is a positive measure of values for η such that $\phi_{NL}^{*}\left(\sigma,\eta\right)<\overline{\pi}$ and a positive measure of values for η such that $\phi_{NL}^{*}\left(\sigma,\eta\right)>\underline{\pi}$, then $\phi_{NL}^{*}\left(\sigma,\eta\right)>\phi_{\theta}^{*}\left(\sigma,\eta\right)$ for a positive measure of values for η. From this result, one can then conclude that, holding σ fixed, the cartel rate is strictly lower with a leniency program.

Holding Y fixed, the threshold function for stable collusion is lower with a leniency program:

$$\phi_{NL}\left(Y,\sigma,\eta\right)-\phi_{\theta}\left(Y,\sigma,\eta\right) \tag{24}$$

$$=\frac{\delta\left(1-\sigma\right)\left(1-\kappa\right)\left(Y-\alpha\mu\right)}{\left(\eta-1\right)\left[1-\delta\left(1-\kappa\right)\right]}$$

$$-\left(\frac{\delta\left(1-\sigma\right)\left(1-\kappa\right)\left(Y-\alpha\mu\right)-\left[1-\delta\left(1-\kappa\right)\right]\left[\sigma-\min\left\{\sigma,\theta\right\}\right]\gamma\left(Y-\alpha\mu\right)}{\left(\eta-1\right)\left[1-\delta\left(1-\kappa\right)\right]}\right)$$

$$=\frac{\left(\sigma-\theta\right)\gamma\left(Y-\alpha\mu\right)}{\eta-1}>0$$

because $\sigma>\theta$. Using $\phi_{NL}\left(Y,\eta\right)>\phi_{\theta}\left(Y,\eta\right)$,

$$\psi_{NL}\left(Y,\sigma,\eta\right)-\psi_{\theta}\left(Y,\sigma,\eta\right)$$

$$=\int_{\underline{\pi}}^{\phi_{NL}\left(Y,\sigma,\eta\right)}\left\{\left(1-\delta\right)\pi+\delta\left[\left(1-\sigma\right)Y+\sigma W\right]-\left(1-\delta\right)\sigma\gamma\left(Y-\alpha\mu\right)\right\}h\left(\pi\right)d\pi$$

$$+\int_{\phi_{NL}\left(Y,\sigma,\eta\right)}^{\overline{\pi}}\left[\left(1-\delta\right)\alpha\pi+\delta W-\left(1-\delta\right)\sigma\gamma\left(Y-\alpha\mu\right)\right]h\left(\pi\right)d\pi$$

$$-\int_{\underline{\pi}}^{\phi_{\theta}\left(Y,\sigma,\eta\right)}\left\{\left(1-\delta\right)\pi+\delta\left[\left(1-\sigma\right)Y+\sigma W\right]-\left(1-\delta\right)\sigma\gamma\left(Y-\alpha\mu\right)\right\}h\left(\pi\right)d\pi$$

$$-\int_{\phi_{\theta}\left(Y,\sigma,\eta\right)}^{\overline{\pi}}\left[\left(1-\delta\right)\alpha\pi+\delta W-\omega\gamma\left(Y-\alpha\mu\right)\right]h\left(\pi\right)d\pi$$

$$=\int_{\phi_{\theta}\left(Y,\sigma,\eta\right)}^{\phi_{NL}\left(Y,\sigma,\eta\right)}\left\{\left(1-\delta\right)\pi+\delta\left[\left(1-\sigma\right)Y+\sigma W\right]\right\}h\left(\pi\right)d\pi-\int_{\phi_{\theta}\left(Y,\sigma,\eta\right)}^{\phi_{NL}\left(Y,\sigma,\eta\right)}\left(1-\delta\right)\sigma\gamma\left(Y-\alpha\mu\right)h\left(\pi\right)d\pi$$

$$+\int_{\phi_{NL}\left(Y,\sigma,\eta\right)}^{\overline{\pi}}\left[\left(1-\delta\right)\alpha\pi+\delta W\right]h\left(\pi\right)d\pi-\int_{\phi_{NL}\left(Y,\sigma,\eta\right)}^{\overline{\pi}}\left(1-\delta\right)\sigma\gamma\left(Y-\alpha\mu\right)h\left(\pi\right)d\pi$$

$$-\int_{\phi_{\theta}\left(Y,\sigma,\eta\right)}^{\overline{\pi}}\left[\left(1-\delta\right)\alpha\pi+\delta W\right]h\left(\pi\right)d\pi+\int_{\phi_{\theta}\left(Y,\sigma,\eta\right)}^{\overline{\pi}}\omega\gamma\left(Y-\alpha\mu\right)h\left(\pi\right)d\pi$$

$$=\int_{\phi_{\theta}\left(Y,\sigma,\eta\right)}^{\phi_{NL}\left(Y,\sigma,\eta\right)}\left\{\left(1-\delta\right)\pi+\delta\left[\left(1-\sigma\right)Y+\sigma W\right]\right\}h\left(\pi\right)d\pi-\int_{\phi_{\theta}\left(Y,\sigma,\eta\right)}^{\phi_{NL}\left(Y,\sigma,\eta\right)}\left[\left(1-\delta\right)\alpha\pi+\delta W\right]h\left(\pi\right)d\pi$$

$$-\int_{\phi_{\theta}\left(Y,\sigma,\eta\right)}^{\overline{\pi}}\left(1-\delta\right)\sigma\gamma\left(Y-\alpha\mu\right)h\left(\pi\right)d\pi+\int_{\phi_{\theta}\left(Y,\sigma,\eta\right)}^{\overline{\pi}}\omega\gamma\left(Y-\alpha\mu\right)h\left(\pi\right)d\pi$$

$$\psi_{NL}(Y,\sigma,\eta) - \psi_\theta(Y,\sigma,\eta) \tag{25}$$

$$= \int_{\phi_\theta(Y,\sigma,\eta)}^{\phi_{NL}(Y,\sigma,\eta)} [(1-\delta)(1-\alpha)\pi + \delta(1-\sigma)(Y-W)]\, h(\pi)\, d\pi$$

$$+ \int_{\phi_\theta(Y,\sigma,\eta)}^{\overline{\pi}} (1-\delta)(\omega-\sigma)\gamma(Y-\alpha\mu)\, h(\pi)\, d\pi.$$

Given $\omega > \sigma$, (25) is non-negative. If $\underline{\pi} \geq \phi_{NL}(Y,\sigma,\eta)$ $(> \phi_\theta(Y,\sigma,\eta))$ or $(\phi_{NL}(Y,\sigma,\eta) >) \phi_\theta(Y,\sigma,\eta) \geq \overline{\pi}$ then the first of the two terms in (25) is zero; otherwise, it is positive. If $\phi_\theta(Y,\sigma,\eta) \geq \overline{\pi}$ then the second term is zero; otherwise, it is positive.

Since it has just been shown that $\psi_{NL}(Y,\sigma,\eta) \geq \psi_\theta(Y,\sigma,\eta)$ then $Y^*_{NL}(\sigma,\eta) \geq Y^*_\theta(\sigma,\eta)$. Given

$$\phi^*(\sigma,\eta) \equiv \max\{\min\{\phi(Y^*(\sigma,\eta),\sigma,\eta),\overline{\pi}\},\underline{\pi}\},$$

it follows that $\phi^*_{NL}(\sigma,\eta) \geq \phi^*_\theta(\sigma,\eta)$.

Next we want to show: if there is positive measure of values for η such that $\phi^*_{NL}(\sigma,\eta) < \overline{\pi}$ and a positive measure of values for η such that $\phi^*_{NL}(\sigma,\eta) > \underline{\pi}$ then $\phi^*_{NL}(\sigma,\eta) > \phi^*_\theta(\sigma,\eta)$ for a positive measure of values for η. If $\phi^*_{NL}(\sigma,\eta) < \overline{\pi}$ then either $\phi^*_{NL}(\sigma,\eta) > \underline{\pi}$ - so that $\phi^*_{NL}(\sigma,\eta) \in (\underline{\pi},\overline{\pi})$ - or $\phi^*_{NL}(\sigma,\eta) = \underline{\pi}$; and if $\phi^*_{NL}(\sigma,\eta) > \underline{\pi}$ then either $\phi^*_{NL}(\sigma,\eta) < \overline{\pi}$ - so that $\phi^*_{NL}(\sigma,\eta) \in (\underline{\pi},\overline{\pi})$ - or $\phi^*_{NL}(\sigma,\eta) = \overline{\pi}$. This results in two mutually exclusive cases: 1) there is a positive measure of values for η such that $\phi^*_{NL}(\sigma,\eta) \in (\underline{\pi},\overline{\pi})$; and 2) there is not a positive measure of values for η such that $\phi^*_{NL}(\sigma,\eta) \in (\underline{\pi},\overline{\pi})$ in which case there is a positive measure of values for η such that $\phi^*_{NL}(\sigma,\eta) = \underline{\pi}$ and a positive measure of values for η such that $\phi^*_{NL}(\sigma,\eta) = \overline{\pi}$.

In considering case (1), first note that

$$\phi^*_{NL}(\sigma,\eta) = \phi(Y^*_{NL}(\sigma,\eta),\sigma,\eta) > \phi_\theta(Y^*_{NL}(\sigma,\eta),\sigma,\eta) \geq \phi_\theta(Y^*_\theta(\sigma,\eta),\sigma,\eta), \tag{26}$$

where the equality follows from $\phi^*_{NL}(\sigma,\eta) \in (\underline{\pi},\overline{\pi})$, the strict inequality follows from $\phi_{NL}(Y,\sigma,\eta) > \phi_\theta(Y,\sigma,\eta)$, and the weak inequality follows from $Y^*_{NL}(\sigma,\eta) \geq Y^*_\theta(\sigma,\eta)$. (26) implies $\overline{\pi} > \phi_\theta(Y^*_\theta(\sigma,\eta),\sigma,\eta)$ and, therefore,

$$\phi^*_\theta(\sigma,\eta) = \max\{\phi_\theta(Y^*_\theta(\sigma,\eta),\sigma,\eta),\underline{\pi}\}. \tag{27}$$

(26)-(27) allow us to conclude: $\phi^*_{NL}(\sigma,\eta) > \phi^*_\theta(\sigma,\eta)$. Hence, for case (1), there is a positive measure of values for η for which $\phi^*_{NL}(\sigma,\eta) > \phi^*_\theta(\sigma,\eta)$. Under case (2), that $\phi^*_{NL}(\sigma,\eta)$ is weakly decreasing in (Lemma 2) implies $\exists \widehat{\eta}_{NL} \in (\underline{\eta},\overline{\eta})$ such that

$$\phi^*_{NL}(\sigma,\eta) = \begin{cases} \overline{\pi} & \text{if } \eta \in [\underline{\eta},\widehat{\eta}_{NL}] \\ \underline{\pi} & \text{if } \eta \in (\widehat{\eta}_{NL},\overline{\eta}] \end{cases}. \tag{28}$$

Note that, at the critical value $\widehat{\eta}_{NL}$,

$$\psi_{NL}(Y,\sigma,\widehat{\eta}_{NL}) \leq Y, \ \forall Y \in [\alpha\mu,\mu], \tag{29}$$

for suppose not. Then $\exists Y' \in (\alpha\mu, \mu)$ such that $\psi_{NL}(Y', \sigma, \widehat{\eta}_{NL}) > Y'$. By the continuity of ψ_{NL} in η, $\exists \xi > 0$ such that $\psi_{NL}(Y', \sigma, \widehat{\eta}_{NL} + \xi) > Y'$ which implies $Y_{NL}^*(\sigma, \widehat{\eta}_{NL} + \xi) > \alpha\mu$ and $\phi_{NL}^*(\sigma, \widehat{\eta}_{NL} + \xi) > \underline{\pi}$, but that contradicts (28). With (29) and $\psi_{NL}(Y, \sigma, \eta) > \psi_\theta(Y, \sigma, \eta)$, it follows $\exists \chi > 0$ such that $\psi_\theta(Y, \sigma, \widehat{\eta}_{NL}) < Y - \chi$ $\forall Y \in [\alpha\mu, \mu]$ which implies, by the continuity of ψ_θ in η, $\exists \widehat{\eta}_\theta < \widehat{\eta}_{NL}$ such that $\phi_\theta^*(\sigma, \eta) = \underline{\pi}$ iff $\eta > \widehat{\eta}_\theta$. We then have that there is a positive measure of values of η - specifically, $\eta \in [\widehat{\eta}_{NL}, \widehat{\eta}_\theta)$ - for which

$$\phi_{NL}^*(\sigma, \eta) = \overline{\pi} > \underline{\pi} = \phi_\theta^*(\sigma, \eta).$$

This concludes the proof that: if there is positive measure of values for η such that $\phi_{NL}^*(\sigma, \eta) < \overline{\pi}$ and a positive measure of values for η such that $\phi_{NL}^*(\sigma, \eta) > \underline{\pi}$ then $\phi_{NL}^*(\sigma, \eta) > \phi_\theta^*(\sigma, \eta)$ for positive measure of values for η.

Whether with or without a leniency program, if the threshold for a type-η industry is $\widetilde{\phi}(\sigma, \eta)$ then the cartel rate is

$$\int_{\underline{\eta}}^{\overline{\eta}} \left[\frac{\kappa(1-\sigma) H\left(\widetilde{\phi}(\sigma, \eta)\right)}{1 - (1-\kappa)(1-\sigma) H\left(\widetilde{\phi}(\sigma, \eta)\right)} \right] g(\eta) d\eta. \tag{30}$$

Note that the cartel rate is increasing in $\widetilde{\phi}(\sigma, \eta)$. Given it has been shown $\phi_{NL}^*(\sigma, \eta) \geq \phi_\theta^*(\sigma, \eta)$ $\forall \eta$, (30) implies $C_{NL}(\sigma) \geq C_\theta(\sigma)$. It has also been shown that: if there is a positive measure of values of η such that $\phi_{NL}^*(\sigma, \eta) < \overline{\pi}$ and a positive measure of values for η such that $\phi_{NL}^*(\sigma, \eta) > \underline{\pi}$ then there is a positive measure of values of η such that $\phi_{NL}^*(\sigma, \eta) > \phi_\theta^*(\sigma, \eta)$ and, therefore,

$$\frac{\kappa(1-\sigma) H(\phi_{NL}^*(\sigma, \eta))}{1 - (1-\kappa)(1-\sigma) H(\phi_{NL}^*(\sigma, \eta))} > \frac{\kappa(1-\sigma) H(\phi_\theta^*(\sigma, \eta))}{1 - (1-\kappa)(1-\sigma) H(\phi_\theta^*(\sigma, \eta))}. \tag{31}$$

As (31) holds for a positive measure of values of η, (30) implies $C_{NL}(\sigma) > C_\theta(\sigma)$. $\blacksquare$

Proof of Theorem 7. When H is uniform, we will show that a type-η industry either is able to collude for all market conditions - $\phi_{NL}^*(\sigma, \eta) = \overline{\pi}$ - or is unable to collude for all market conditions - $\phi_{NL}^*(\sigma, \eta) = \underline{\pi}$. Hence, $\exists \widehat{\eta}_{NL}$ such that if $\eta \leq \widehat{\eta}_{NL}$ then collusion is always stable, and if $\eta > \widehat{\eta}_{NL}$ then collusion is never stable. A leniency program is shown to lower this threshold value and from that result it will be shown that introducing a leniency program raises the equilibrium value for σ and lowers the equilibrium cartel rate. The proof involves some tedious calculations associated with deriving the derivatives of $\psi_{NL}(Y)$ and solving for the threshold values; those calculations can be found in Appendix B.

Under the assumptions that H is uniform and γ is sufficiently close to zero, if

there is no leniency program then it is shown in Appendix B that

$$
\psi'_{NL}(Y) = \begin{cases}
\frac{\delta\kappa}{1-\delta(1-\kappa)} - (1-\delta)\sigma\gamma & \text{if } Y \leq A \\[2ex]
[(1-\delta)(1-\alpha)\phi_{NL}(Y) + \delta(1-\sigma)(Y-W)]\,h(\phi_{NL}(Y))\,\phi'_{NL}(Y) & \text{if } Y \in (A,B) \\[1ex]
\quad + \left[\delta(1-\sigma) + \delta\sigma\frac{\partial W}{\partial Y}\right]H(\phi_{NL}(Y)) & \\[1ex]
\quad + \delta\frac{\partial W}{\partial Y}[1 - H(\phi_{NL}(Y))] - (1-\delta)\sigma\gamma & \\[2ex]
\delta\left(\frac{1-\delta(1-\kappa)-\sigma(1-\kappa)(1-\delta)}{1-\delta(1-\kappa)}\right) - (1-\delta)\sigma\gamma & \text{if } Y \geq B
\end{cases}
$$

where

$$
\phi'_{NL}(Y) = \frac{\delta(1-\sigma)(1-\kappa)}{(\eta-1)[1-\delta(1-\kappa)]}
$$

$$
A \equiv \alpha\mu + \frac{\underline{\pi}(\eta-1)[1-\delta(1-\kappa)]}{\delta(1-\sigma)(1-\kappa)}
$$

$$
B \equiv \alpha\mu + \frac{\overline{\pi}(\eta-1)[1-\delta(1-\kappa)]}{\delta(1-\sigma)(1-\kappa)}
$$

When $Y \leq A$, $\phi_{NL}(Y) = \underline{\pi}$ so collusion is not stable for all market conditions; when $Y \in (A,B)$ then $\phi_{NL}(Y) \in (\underline{\pi}, \overline{\pi})$ so collusion is stable for some market conditions; and when $Y \geq B$ then $\phi_{NL}(Y) = \overline{\pi}$ so collusion is stable for all market conditions. Note that, when $\gamma \simeq 0$, $\psi'_{NL}(Y) \in (0,1)$ if $Y \leq A$ or $Y \geq B$. $\psi_{NL}(Y)$ is linear when Y does not affect $\phi_{NL}(Y)$ and is quadratic when it does:

$$
\psi''_{NL}(Y) = \begin{cases}
0 & \text{if } Y \leq A \\[1ex]
\left[(1-\delta)(1-\alpha)\phi'_{NL}(Y) + 2\delta(1-\sigma)\left(1-\frac{\partial W}{\partial Y}\right)\right]h(\phi_{NL}(Y))\,\phi'_{NL}(Y) & \text{if } Y \in (A,B) \\[1ex]
0 & \text{if } Y \geq B
\end{cases}
$$

$$
\psi'''_{NL}(Y) = 0, \ \forall Y.
$$

If $A \geq \mu$ then ψ_{NL} is linear and $\psi'_{NL}(Y) \in (0,1)$, $\forall Y \in [\alpha\mu, \mu]$; see Figure 7a. Given that $\psi_{NL}(\alpha\mu) = \alpha\mu$, it follows that $\psi_{NL}(Y) < Y \ \forall Y > \alpha\mu$ which implies that $\alpha\mu$ is the unique fixed point of $\alpha\mu$; this industry type never colludes. If $B \geq \mu$ then again there is a unique fixed point of $\alpha\mu$ by the following argument. When $B \geq \mu$, ψ_{NL} is weakly convex $\forall Y \in [\alpha\mu, \mu]$ (it is linear then strictly convex); see Figure 7b. Given that $\psi_{NL}(\alpha\mu) = \alpha\mu$ and $\psi_{NL}(\mu) < \mu$ then $\psi_{NL}(Y) < Y \forall Y \in (\alpha\mu, \mu]$. Thus, a necessary condition for there to be a fixed point exceeding $\alpha\mu$ is that $B < \mu$. Note that B is increasing in η and $\lim_{\eta \to 1} B = \alpha\mu$. Thus, if η is sufficiently low then $B < \mu$. Values of η such that $B < \mu$ exist by supposition because it was presumed that the equilibrium cartel rate without a leniency program is positive which means some industry types are able to collude.

Assume $B < \mu$ and $\psi_{NL}(B) < B$; see Figure 7c. Given $\psi_{NL}(\alpha\mu) = \alpha\mu$ and ψ_{NL} is weakly convex $\forall Y \in [\alpha\mu, B]$, $\psi_{NL}(B) < B$ implies $\psi_{NL}(Y) < Y \ \forall Y \in (\alpha\mu, B)$. Next note that for $Y \in [B, \mu]$, ψ_{NL} is linear and $\psi'_{NL}(Y) \in (0, 1)$ in which case $\psi_{NL}(B) < B$ implies $\psi_{NL}(Y) < Y \ \forall Y \in [B, \mu]$. We then have that $\psi_{NL}(B) < B$ implies $\psi_{NL}(Y) < Y \ \forall Y \in (\alpha\mu, \mu]$ which means there is a unique fixed point of $\alpha\mu$.

Finally, assume $B < \mu$ and $\psi_{NL}(B) \geq B$; see Figure 7d. If $\psi_{NL}(B) > B$ then there is a unique fixed point in $(\alpha\mu, B)$ where uniqueness comes from ψ_{NL} being weakly convex over $[\alpha\mu, B]$. Since ψ_{NL} is linear, $\psi'_{NL}(Y) \in (0, 1)$ for $Y \in [B, \mu]$, and $\psi_{NL}(\mu) < \mu$ then there is a second fixed point in (B, μ). Thus, if $\psi_{NL}(B) > B$ then there are two fixed points exceeding $\alpha\mu$. When $\psi_{NL}(B) = B$, there is one fixed point exceeding $\alpha\mu$ which is B. In sum, when $\psi_{NL}(B) \geq B$, the maximal fixed point is at least B which means that it occurs where $\phi_{NL}(Y) = \overline{\pi}$. Using the expression for $\psi_{NL}(Y)$ when $\phi_{NL}(Y) = \overline{\pi}$ (which is the LHS of (32)), the collusive value $Y^*_{NL}(\sigma, \eta)$ is the unique solution to:

$$(1 - \delta)\mu + \delta Y^*_{NL}(\sigma, \eta) - \left[\delta\sigma\left(\frac{(1 - \kappa)(1 - \delta)}{1 - \delta(1 - \kappa)}\right) + (1 - \delta)\sigma\gamma\right](Y^*_{NL}(\sigma, \eta) - \alpha\mu) = Y^*_{NL}(\sigma, \eta).$$

(32)

Summarizing, if η is such that $B \geq \mu$ or $B < \mu$ and $\psi_{NL}(B) < B$ then collusion is unstable for all market conditions so $\phi^*_{NL}(\sigma, \eta) = \underline{\pi}$. If $B < \mu$ and $\psi_{NL}(B) \geq B$ then collusion is stable for all market conditions so $\phi^*_{NL}(\sigma, \eta) = \overline{\pi}$. Hence, $\widehat{\eta}_{NL}(\sigma)$ is the lowest value for η such that $\phi^*_{NL}(\sigma, \eta) = \overline{\pi}$. Given ψ_{NL} is continuous, $\widehat{\eta}_{NL}(\sigma)$ is defined by $\psi_{NL}(B, \sigma, \widehat{\eta}_{NL}) = B$, which takes the explicit form:

$$(1 - \delta)\mu + \delta B - \left[\delta\sigma\left(\frac{(1 - \kappa)(1 - \delta)}{1 - \delta(1 - \kappa)}\right) + (1 - \delta)\sigma\gamma\right](B - \alpha\mu) = B.$$

Substituting for B and solving for $\widehat{\eta}_{NL}(\sigma)$ (derivations are in the appendix),

$$\widehat{\eta}_{NL}(\sigma) = 1 + \left(\frac{\mu}{\overline{\pi}}\right)\left(\frac{\delta(1 - \sigma)(1 - \kappa)(1 - \alpha)}{\delta\sigma(1 - \kappa) + (1 + \sigma\gamma)(1 - \delta(1 - \kappa))}\right) \tag{33}$$

Without a leniency program, cartels then emerge only in industries for which $\eta \leq \widehat{\eta}_{NL}(\sigma)$ and those cartels never internally collapse though are shut down by the CA at a rate of σ per period.

We now need to repeat the analysis for when there is a full leniency program $(\theta = 0)$. The steps are exactly the same as above; it is just that some expressions are different.

$$\psi'_L(Y) = \begin{cases} \frac{\delta\kappa}{1 - \delta(1 - \kappa)} - (1 - \delta)\omega\gamma & \text{if } Y \leq C \\[2ex] [(1 - \delta)(1 - \alpha)\phi_L(Y) + \delta(1 - \sigma)(Y - W) - (1 - \delta)(\sigma - \omega)\gamma(Y - \alpha\mu)] \times & \text{if } Y \in (C, D) \\ h(\phi_L(Y))\phi'_L(Y) + \left[\delta(1 - \sigma) + \delta\sigma\frac{\partial W}{\partial Y} - (1 - \delta)\sigma\gamma\right]H(\phi_L(Y)) \\ + \left[\delta\frac{\partial W}{\partial Y} - (1 - \delta)\omega\gamma\right](1 - H(\phi_L(Y))) \\[2ex] \delta\left(1 - \sigma\left(\frac{(1 - \kappa)(1 - \delta)}{1 - \delta(1 - \kappa)}\right)\right) - (1 - \delta)\sigma\gamma & \text{if } Y \geq D \end{cases}$$

where

$$\phi_L'(Y) = \frac{\delta(1-\sigma)(1-\kappa) - [1-\delta(1-\kappa)](\sigma-\theta)\gamma}{(\eta-1)[1-\delta(1-\kappa)]}$$

$$C \equiv \alpha\mu + \frac{\underline{\pi}(\eta-1)[1-\delta(1-\kappa)]}{\delta(1-\sigma)(1-\kappa) - [1-\delta(1-\kappa)](\sigma-\theta)\gamma}$$

$$D \equiv \alpha\mu + \frac{\overline{\pi}(\eta-1)[1-\delta(1-\kappa)]}{\delta(1-\sigma)(1-\kappa) - [1-\delta(1-\kappa)](\sigma-\theta)\gamma}$$

Furthermore, $\psi_L'(Y) \in (0,1)$ if $Y \leq C$ or $Y \geq D$.

$$\psi_L''(Y) = \begin{cases} 0 & \text{if } Y \leq C \\[2ex] \left[(1-\delta)(1-\alpha)\phi_L'(Y) + 2\delta(1-\sigma)\left(1 - \frac{\partial W}{\partial Y}\right) - 2(1-\delta)(\sigma-\omega)\gamma\right] \times & \text{if } Y \in (C,D) \\ h(\phi_L(Y))\phi_L'(Y) & \\[2ex] 0 & \text{if } Y \geq D \end{cases}$$

$$\psi_L'''(Y) = 0, \ \forall Y.$$

Again, ψ_L is a linear then quadratic then linear function.

If $C \geq \mu$ then there is a unique fixed point of $\alpha\mu$, in which case this industry type never cartelizes. If $D \geq \mu$ then again there is a unique fixed point of $\alpha\mu$. Thus, a necessary condition for there to be a fixed point exceeding $\alpha\mu$ is that $D < \mu$. Assume η is sufficiently low so that $D < \mu$. If no values for η in $[\underline{\eta}, \overline{\eta}]$ exist whereby $D < \mu$, it means that all industry types cannot collude for all market conditions; hence, the cartel rate with a leniency program is zero, which proves that it is lower than when there is no leniency program. Thus, let us suppose that there are values for η such that $D < \mu$.

If $\psi_L(D) < D$ then, as argued for the case of no leniency program, there are no fixed points exceeding $\alpha\mu$. If $\psi_L(D) > (=)D$ then there are two (one) fixed points exceeding $\alpha\mu$ and the maximal fixed point is at least D which means that it occurs where $\phi_L(Y) = \overline{\pi}$. Thus, if η is such that $D \geq \mu$ or $D < \mu$ and $\psi_L(D) < D$ then collusion is unstable for all market conditions so $\phi_L^*(\sigma,\eta) = \underline{\pi}$. If $D < \mu$ and $\psi_L(D) \geq D$ then collusion is stable for all market conditions so $\phi_L^*(\sigma,\eta) = \overline{\pi}$. $\widehat{\eta}_L(\sigma)$ is then the highest value for η such that $\psi_L(D) \geq D$ which, by the continuity of ψ_L, is that value for η such that $\psi_L(D) = D$. Given $\phi_L(Y) = \overline{\pi}$, $\psi_L(D) = D$ takes the form:

$$(1-\delta)\mu + \delta D - \delta\sigma\left(\frac{(1-\kappa)(1-\delta)(D-\alpha\mu)}{1-\delta(1-\kappa)}\right) - (1-\delta)\sigma\gamma(D-\alpha\mu) = D.$$

Substituting for D and solving for $\widehat{\eta}_L(\sigma)$ yields:

$$\widehat{\eta}_L(\sigma) = 1 + \left(\frac{\mu}{\overline{\pi}}\right)\left(\frac{(1-\alpha)(\delta(1-\sigma)(1-\kappa) - (1-\delta(1-\kappa))\sigma\gamma)}{\delta\sigma(1-\kappa) + (1+\sigma\gamma)(1-\delta(1-\kappa))}\right) \qquad (34)$$

Let us now combine the analyses for the cases of no leniency program and a full leniency program. For when there is no leniency program, a type-η cartel colludes for all market conditions when $\eta \leq \widehat{\eta}_{NL}(\sigma)$, and does not collude for all market conditions when $\eta > \widehat{\eta}_{NL}(\sigma)$. For when there is a full leniency program, a type-η cartel colludes for all market conditions when $\eta \leq \widehat{\eta}_L(\sigma)$, and does not collude for all market conditions when $\eta > \widehat{\eta}_L(\sigma)$. Using the expressions in (33) and (34),

$$\widehat{\eta}_{NL}(\sigma) - \widehat{\eta}_L(\sigma) = \left(\frac{\mu}{\overline{\pi}}\right)\left(\frac{(1 - \delta(1 - \kappa))\sigma\gamma}{\delta\sigma(1 - \kappa) + (1 + \sigma\gamma)(1 - \delta(1 - \kappa))}\right) > 0.$$

Hence, holding σ fixed, fewer industry types cartelize when there is a leniency program.

Whether or not there is a leniency program, the formula for the cartel rate is

$$\int_{\underline{\eta}}^{\overline{\eta}}\left[\frac{\kappa(1 - \sigma)H\left(\widetilde{\phi}(\sigma, \eta)\right)}{1 - (1 - \kappa)(1 - \sigma)H\left(\widetilde{\phi}(\sigma, \eta)\right)}\right]g(\eta)\,d\eta,$$

where $\widetilde{\phi}(\sigma, \eta) = \phi_{NL}^*(\sigma, \eta)$ without a leniency program, and $\widetilde{\phi}(\sigma, \eta) = \phi_L^*(\sigma, \eta)$ with a leniency program. Given that

$$\phi_{NL}^*(\sigma, \eta) = \begin{cases} \overline{\pi} & \text{if } \eta \leq \widehat{\eta}_{NL}(\sigma) \\ \underline{\pi} & \text{if } \eta > \widehat{\eta}_{NL}(\sigma) \end{cases}$$

then the cartel rate function without a leniency program is

$$C_{NL}(\sigma) = \int_{\underline{\eta}}^{\widehat{\eta}_{NL}(\sigma)}\left[\frac{\kappa(1 - \sigma)}{1 - (1 - \kappa)(1 - \sigma)}\right]g(\eta)\,d\eta = \frac{\kappa(1 - \sigma)G(\widehat{\eta}_{NL}(\sigma))}{1 - (1 - \kappa)(1 - \sigma)}.$$

Given that

$$\phi_L^*(\sigma, \eta) = \begin{cases} \overline{\pi} & \text{if } \eta \leq \widehat{\eta}_L(\sigma) \\ \underline{\pi} & \text{if } \eta > \widehat{\eta}_L(\sigma) \end{cases}$$

then the cartel rate function with a leniency program is

$$C_L(\sigma) = \int_{\underline{\eta}}^{\widehat{\eta}_L(\sigma)}\left[\frac{\kappa(1 - \sigma)}{1 - (1 - \kappa)(1 - \sigma)}\right]g(\eta)\,d\eta = \frac{\kappa(1 - \sigma)G(\widehat{\eta}_L(\sigma))}{1 - (1 - \kappa)(1 - \sigma)}.$$

Thus, holding σ fixed, $\widehat{\eta}_{NL}(\sigma) > \widehat{\eta}_L(\sigma)$ implies that a leniency program reduces the cartel rate:

$$C_{NL}(\sigma) = \frac{\kappa(1 - \sigma)G(\widehat{\eta}_{NL}(\sigma))}{1 - (1 - \kappa)(1 - \sigma)} > \frac{\kappa(1 - \sigma)G(\widehat{\eta}_L(\sigma))}{1 - (1 - \kappa)(1 - \sigma)} = C_L(\sigma). \qquad (35)$$

The final step is to endogenize σ. Without a leniency program, σ_{NL}^* is defined by:

$$\sigma_{NL}^* = qrp\left(\frac{qr\kappa(1 - \sigma_{NL}^*)G(\widehat{\eta}_{NL}(\sigma_{NL}^*))}{1 - (1 - \kappa)(1 - \sigma_{NL}^*)}\right),$$

and, given that it is the maximal fixed point,

$$\sigma \gtreqqless qrp\left(\frac{qr\kappa\left(1-\sigma\right)G\left(\widehat{\eta}_{NL}\left(\sigma\right)\right)}{1-\left(1-\kappa\right)\left(1-\sigma\right)}\right) \quad \text{as } \sigma \gtreqqless \sigma_{NL}^{*}. \tag{36}$$

Since cartels never collapse - they either form and are always stable or do not form - then there are no leniency applications (recall that only dying cartels apply for leniency). As a result, the equilibrium cartel rate with a leniency program σ_L^* is

$$\sigma_L^* = qrp\left(\frac{qr\kappa\left(1-\sigma_L^*\right)G\left(\widehat{\eta}_L\left(\sigma_L^*\right)\right)}{1-\left(1-\kappa\right)\left(1-\sigma_L^*\right)}\right). \tag{37}$$

Given p is decreasing, it follows from (35):

$$qrp\left(\frac{qr\kappa\left(1-\sigma\right)G\left(\widehat{\eta}_L\left(\sigma\right)\right)}{1-\left(1-\kappa\right)\left(1-\sigma\right)}\right) > qrp\left(\frac{qr\kappa\left(1-\sigma\right)G\left(\widehat{\eta}_{NL}\left(\sigma\right)\right)}{1-\left(1-\kappa\right)\left(1-\sigma\right)}\right). \tag{38}$$

Using (36), (38) implies

$$qrp\left(\frac{qr\kappa\left(1-\sigma_{NL}^*\right)G\left(\widehat{\eta}_L\left(\sigma_{NL}^*\right)\right)}{1-\left(1-\kappa\right)\left(1-\sigma_{NL}^*\right)}\right) > \sigma_{NL}^*, \tag{39}$$

and, therefore, $\sigma_L^* > \sigma_{NL}^*$. Given $C_{NL}\left(\sigma\right) > C_L\left(\sigma\right)$ from (35), and $C_L\left(\sigma\right)$ is decreasing in σ (Lemma 4), $\sigma_L^* > \sigma_{NL}^*$ implies the equilibrium cartel rate is lower with a leniency program:

$$C_{NL}\left(\sigma_{NL}^*\right) > C_L\left(\sigma_{NL}^*\right) > C_L\left(\sigma_L^*\right).$$

∎

Proof of Theorem 8. Given $\sigma^* \in (0,\omega)$ and $\theta = 0$, by Theorem 6 we have that $C_{NL}\left(\sigma\right) \geq C_L\left(\sigma\right)$ and, when there is positive measure of values for η such that $\phi_{NL}^*\left(\sigma,\eta\right) < \overline{\pi}$ and a positive measure of values for η such that $\phi_{NL}^*\left(\sigma,\eta\right) > \underline{\pi}$, $C_{NL}\left(\sigma\right) > C_L\left(\sigma\right)$. To prove this theorem, it is then sufficient to show $\sigma_L^* > \sigma_{NL}^*$.

σ_{NL}^* and σ_L^* are defined by:

$$\sigma_{NL}^* = qrp\left(qr\int_{\underline{\eta}}^{\overline{\eta}} C_{NL}\left(\sigma_{NL}^*,\eta\right)g\left(\eta\right)d\eta\right)$$

where

$$C_{NL}\left(\sigma,\eta\right) = \frac{\kappa\left(1-\sigma\right)H\left(\phi_{NL}^*\left(\sigma,\eta\right)\right)}{1-\left(1-\kappa\right)\left(1-\sigma\right)H\left(\phi_{NL}^*\left(\sigma,\eta\right)\right)},$$

and

$$\begin{aligned}
\sigma_L^* &= p\left(\lambda\int_{\underline{\eta}}^{\overline{\eta}}\left(1-H\left(\phi_L^*\left(\sigma_L^*,\eta\right)\right)\right)C_L\left(\sigma_L^*,\eta\right)g\left(\eta\right)d\eta\right. \\
&\quad \left. +qr\int_{\underline{\eta}}^{\overline{\eta}}H\left(\phi_L^*\left(\sigma_L^*,\eta\right)\right)C_L\left(\sigma_L^*,\eta\right)g\left(\eta\right)d\eta\right)
\end{aligned}$$

46

where
$$C_L\left(\sigma,\eta\right)=\frac{\kappa\left(1-\sigma\right)H\left(\phi_L^*\left(\sigma,\eta\right)\right)}{1-\left(1-\kappa\right)\left(1-\sigma\right)H\left(\phi_L^*\left(\sigma,\eta\right)\right)}.$$

If $\phi_{NL}^*\left(\sigma,\eta\right)>\left(=\right)\phi_L^*\left(\sigma,\eta\right)$ then

$$H\left(\phi_{NL}^*\left(\sigma,\eta\right)\right)>\left(=\right)H\left(\phi_L^*\left(\sigma,\eta\right)\right)$$

and

$$C_{NL}\left(\sigma,\eta\right)>\left(=\right)C_L\left(\sigma,\eta\right),$$

in which case,

$$H\left(\phi_{NL}^*\left(\sigma,\eta\right)\right)C_{NL}\left(\sigma,\eta\right)>\left(=\right)H\left(\phi_L^*\left(\sigma,\eta\right)\right)C_L\left(\sigma,\eta\right).$$

It is immediate that if

$$H\left(\phi_{NL}^*\left(\sigma,\eta\right)\right)C_{NL}\left(\sigma,\eta\right)\geq H\left(\phi_L^*\left(\sigma,\eta\right)\right)C_L\left(\sigma,\eta\right),\forall\eta \tag{40}$$

and

$$H\left(\phi_{NL}^*\left(\sigma,\eta\right)\right)C_{NL}\left(\sigma,\eta\right)>H\left(\phi_L^*\left(\sigma,\eta\right)\right)C_L\left(\sigma,\eta\right),\text{ for positive measure of }\eta \tag{41}$$

then

$$\int_{\underline{\eta}}^{\overline{\eta}}H\left(\phi_{NL}^*\left(\sigma,\eta\right)\right)C_{NL}\left(\sigma,\eta\right)g\left(\eta\right)d\eta>\int_{\underline{\eta}}^{\overline{\eta}}H\left(\phi_L^*\left(\sigma,\eta\right)\right)C_L\left(\sigma,\eta\right)g\left(\eta\right)d\eta. \tag{42}$$

(40) is always true and (41) is true when there is positive measure of values for η such that $\phi_{NL}^*\left(\sigma,\eta\right)<\overline{\pi}$ and a positive measure of values for η such that $\phi_{NL}^*\left(\sigma,\eta\right)>\underline{\pi}$.

Evaluate (42) at $\sigma=\sigma_{NL}^*$:

$$\int_{\underline{\eta}}^{\overline{\eta}}H\left(\phi_{NL}^*\left(\sigma_{NL}^*,\eta\right)\right)C_{NL}\left(\sigma_{NL}^*,\eta\right)g\left(\eta\right)d\eta>\int_{\underline{\eta}}^{\overline{\eta}}H\left(\phi_L^*\left(\sigma_{NL}^*,\eta\right)\right)C_L\left(\sigma_{NL}^*,\eta\right)g\left(\eta\right)d\eta. \tag{43}$$

Noting that σ_{NL}^* does not depend on λ, if λ is sufficiently small then it follows from (43):

$$qr\int_{\underline{\eta}}^{\overline{\eta}}H\left(\phi_{NL}^*\left(\sigma_{NL}^*,\eta\right)\right)C_{NL}\left(\sigma_{NL}^*,\eta\right)g\left(\eta\right)d\eta \tag{44}$$

$$>\quad\lambda\int_{\underline{\eta}}^{\overline{\eta}}\left(1-H\left(\phi_L^*\left(\sigma_{NL}^*,\eta\right)\right)\right)C_L\left(\sigma_{NL}^*,\eta\right)g\left(\eta\right)d\eta+$$

$$qr\int_{\underline{\eta}}^{\overline{\eta}}H\left(\phi_L^*\left(\sigma_{NL}^*,\eta\right)\right)C_L\left(\sigma_{NL}^*,\eta\right)g\left(\eta\right)d\eta$$

Given that p is decreasing then (44) implies (when λ is sufficiently small):

$$qrp\left(\lambda\int_{\underline{\eta}}^{\overline{\eta}}\left(1-H\left(\phi_L^*\left(\sigma_{NL}^*,\eta\right)\right)\right)C_L\left(\sigma_{NL}^*,\eta\right)g\left(\eta\right)d\eta\right.$$

$$\left.+qr\int_{\underline{\eta}}^{\overline{\eta}}H\left(\phi_L^*\left(\sigma_{NL}^*,\eta\right)\right)C_L\left(\sigma_{NL}^*,\eta\right)g\left(\eta\right)d\eta\right)$$

$$>\quad qrp\left(qr\int_{\underline{\eta}}^{\overline{\eta}}H\left(\phi_{NL}^*\left(\sigma_{NL}^*,\eta\right)\right)C_{NL}\left(\sigma_{NL}^*,\eta\right)g\left(\eta\right)d\eta\right)$$

$$>\quad qrp\left(qr\int_{\underline{\eta}}^{\overline{\eta}}C_{NL}\left(\sigma_{NL}^*,\eta\right)g\left(\eta\right)d\eta\right)=\sigma_{NL}^*.$$

Hence,

$$qrp\left(\lambda\int_{\underline{\eta}}^{\overline{\eta}}\left(1-H\left(\phi_L^*\left(\sigma_{NL}^*,\eta\right)\right)\right)C_L\left(\sigma_{NL}^*,\eta\right)g\left(\eta\right)d\eta\right.$$

$$\left.+qr\int_{\underline{\eta}}^{\overline{\eta}}H\left(\phi_L^*\left(\sigma_{NL}^*,\eta\right)\right)C_L\left(\sigma_{NL}^*,\eta\right)g\left(\eta\right)d\eta\right)$$

$$>\quad \sigma_{NL}^*$$

and thus $\sigma_L^* > \sigma_{NL}^*$.

In proving $\sigma_L^* > \sigma_{NL}^*$, the preceding analysis presumed $\omega > \sigma$. If, contrary to that presumption, $\sigma_L^* \geq \omega$ then the supposition that $\omega > \sigma_{NL}^*$ would again imply $\sigma_L^* > \sigma_{NL}^*$. $\blacksquare$

Proof of Theorem 9. Given that $\sigma = qrs$ and $r,s \in [0,1]$ (hence, are bounded), it is immediate that

$$\lim_{q\to 0}\sigma_{NL}^* = 0,\ \lim_{q\to 0}\sigma_L^* = 0,$$

which implies

$$\lim_{q\to 0}C_{NL}\left(\sigma_{NL}^*\right)=\lim_{\sigma\to 0}C_{NL}\left(\sigma\right),\ \ \lim_{q\to 0}C_L\left(\sigma_L^*\right)=\lim_{\sigma\to 0}C_L\left(\sigma\right).$$

To show the equilibrium cartel rate is lower with a leniency program, it is then sufficient to prove:

$$\lim_{\sigma\to 0}C_{NL}\left(\sigma\right)>\lim_{\sigma\to 0}C_L\left(\sigma\right). \tag{45}$$

Given $\theta = 0 < \omega$ then $\sigma \in (\theta,\omega)$ holds as $\sigma \to 0$ in which case Theorem 6 proves (45). $\blacksquare$

Proof of Theorem 10. The first step is to show that, as the penalty multiple γ goes to zero, the cartel rate function is the same with and without a leniency program:

$$\lim_{\gamma\to 0}C_{NL}\left(\sigma\right)=\lim_{\gamma\to 0}C_L\left(\sigma\right),\ \forall\sigma.$$

48

The second step is to show that, as $\gamma \to 0$, non-leniency enforcement is weaker with a leniency program:

$$\lim_{\gamma \to 0} \sigma_{NL}^* > \lim_{\gamma \to 0} \sigma_L^*.$$

These two results together imply that the equilibrium cartel rate with a leniency program is higher than without a leniency program when $\gamma \simeq 0$.

For the first step, let us begin by considering the thresholds for stable collusion. Without a leniency program,

$$\phi_{NL}(Y, \sigma, \eta) = \frac{\delta (1 - \sigma)(1 - \kappa)(Y - \alpha\mu)}{(\eta - 1)[1 - \delta(1 - \kappa)]}$$

and, trivially,[35]

$$\lim_{\gamma \to 0} \phi_{NL}(Y, \sigma, \eta) = \frac{\delta (1 - \sigma)(1 - \kappa)(Y - \alpha\mu)}{(\eta - 1)[1 - \delta(1 - \kappa)]}.$$

With a full leniency program,

$$\phi_L(Y, \sigma, \eta) = \frac{\delta (1 - \sigma)(1 - \kappa)(Y - \alpha\mu) - [1 - \delta(1 - \kappa)]\sigma\gamma(Y - \alpha\mu)}{(\eta - 1)[1 - \delta(1 - \kappa)]}$$

and

$$\lim_{\gamma \to 0} \phi_L(Y, \sigma, \eta) = \frac{\delta (1 - \sigma)(1 - \kappa)(Y - \alpha\mu)}{(\eta - 1)[1 - \delta(1 - \kappa)]}.$$

Hence,

$$\lim_{\gamma \to 0} \phi_{NL}(Y, \sigma, \eta) = \lim_{\gamma \to 0} \phi_L(Y, \sigma, \eta). \tag{46}$$

Turning to the collusive value functions, we have without a leniency program:

$$
\begin{aligned}
\psi_{NL}(Y, \sigma, \eta) &= \int_{\underline{\pi}}^{\phi_{NL}(Y, \sigma, \eta)} [(1 - \delta)\pi + \delta(1 - \sigma)Y + \delta\sigma W] h(\pi) d\pi \\
&+ \int_{\phi_{NL}(Y, \sigma, \eta)}^{\overline{\pi}} [(1 - \delta)\alpha\pi + \delta W] h(\pi) d\pi - (1 - \delta)\sigma\gamma(Y - \alpha\mu),
\end{aligned}
$$

and with a full leniency program:

$$
\begin{aligned}
\psi_L(Y, \sigma, \eta) &= \int_{\underline{\pi}}^{\phi_L(Y, \sigma, \eta)} [(1 - \delta)\pi + \delta(1 - \sigma)Y + \delta\sigma W - (1 - \delta)\sigma\gamma(Y - \alpha\mu)] h(\pi) d\pi \\
&+ \int_{\phi_L(Y, \sigma, \eta)}^{\overline{\pi}} [(1 - \delta)\alpha\pi + \delta W - (1 - \delta)\omega\gamma(Y - \alpha\mu)] h(\pi) d\pi.
\end{aligned}
$$

---

[35] Recall that $\phi_{NL}(Y, \sigma, \eta)$ comes out of the ICC and is the market condition that makes a firm indifferent between colluding and cheating. That γ does not matter is because the expected penalty is the same whether a firm sets the collusive price or cheats and undercuts the collusive price set by the other firms.

Using (46),

$$
\lim_{\gamma \to 0} \psi_{NL}(Y, \sigma, \eta) \;=\; \lim_{\gamma \to 0} \psi_L(Y, \sigma, \eta) \tag{47}
$$

$$
= \int_{\underline{\pi}}^{\phi_{NL}(Y, \sigma, \eta)} \left[(1 - \delta) \pi + \delta (1 - \sigma) Y + \delta \sigma W \right] h(\pi) \, d\pi
$$

$$
+ \int_{\phi_{NL}(Y, \sigma, \eta)}^{\overline{\pi}} \left[(1 - \delta) \alpha \pi + \delta W \right] h(\pi) \, d\pi.
$$

Generically, (47) implies

$$
\lim_{\gamma \to 0} Y_{NL}^*(\sigma, \eta) = \lim_{\gamma \to 0} Y_L^*(\sigma, \eta). \tag{48}
$$

It follows from (46) and (47) that:

$$
\lim_{\gamma \to 0} \phi_{NL}^*(\sigma, \eta) = \lim_{\gamma \to 0} \phi_L^*(\sigma, \eta). \tag{49}
$$

Given σ, the cartel rate without and with a leniency program, respectively, is:

$$
C_{NL}(\sigma) = \int_{\underline{\eta}}^{\overline{\eta}} C_{NL}(\sigma, \eta) \, g(\eta) \, d\eta = \int_{\underline{\eta}}^{\overline{\eta}} \left[\frac{\kappa (1 - \sigma) H(\phi_{NL}^*(\sigma, \eta))}{1 - (1 - \kappa)(1 - \sigma) H(\phi_{NL}^*(\sigma, \eta))} \right] g(\eta) \, d\eta
$$

$$
C_L(\sigma) = \int_{\underline{\eta}}^{\overline{\eta}} C_L(\sigma, \eta) \, g(\eta) \, d\eta = \int_{\underline{\eta}}^{\overline{\eta}} \left[\frac{\kappa (1 - \sigma) H(\phi_L^*(\sigma, \eta))}{1 - (1 - \kappa)(1 - \sigma) H(\phi_L^*(\sigma, \eta))} \right] g(\eta) \, d\eta.
$$

Using (49),

$$
\lim_{\gamma \to 0} C_{NL}(\sigma) = \lim_{\gamma \to 0} C_L(\sigma). \tag{50}
$$

To prove the second step, we want to first show that, when $\lambda \simeq 1$ and $\gamma \simeq 0$,

$$
p \left(qr \int_{\underline{\eta}}^{\overline{\eta}} C_{NL}(\sigma, \eta) \, g(\eta) \, d\eta \right) \tag{51}
$$

$$
> \; p \left(\lambda \int_{\underline{\eta}}^{\overline{\eta}} (1 - H(\phi_L^*(\sigma, \eta))) C_L(\sigma, \eta) \, g(\eta) \, d\eta + qr \int_{\underline{\eta}}^{\overline{\eta}} H(\phi_L^*(\sigma, \eta)) C_L(\sigma, \eta) \, g(\eta) \, d\eta \right).
$$

Given p is strictly decreasing, (51) holds iff

$$
\lambda \int_{\underline{\eta}}^{\overline{\eta}} (1 - H(\phi_L^*(\sigma, \eta))) C_L(\sigma, \eta) \, g(\eta) \, d\eta + qr \int_{\underline{\eta}}^{\overline{\eta}} H(\phi_L^*(\sigma, \eta)) C_L(\sigma, \eta) \, g(\eta) \, d\eta
$$

$$
> \; qr \int_{\underline{\eta}}^{\overline{\eta}} C_{NL}(\sigma, \eta) \, g(\eta) \, d\eta
$$

or, equivalently,

$$
\int_{\underline{\eta}}^{\overline{\eta}} (1 - H(\phi_L^*(\sigma, \eta))) \left[\lambda C_L(\sigma, \eta) - qr C_{NL}(\sigma, \eta) \right] g(\eta) \, d\eta \tag{52}
$$

$$
> \; qr \int_{\underline{\eta}}^{\overline{\eta}} H(\phi_L^*(\sigma, \eta)) \left[C_{NL}(\sigma, \eta) - C_L(\sigma, \eta) \right] g(\eta) \, d\eta.
$$

50

Given (50), (52) holds as $\gamma \to 0$ iff

$$(\lambda - qr) \int_{\underline{\eta}}^{\overline{\eta}} (1 - H(\phi_{NL}^*(\sigma, \eta))) C_{NL}(\sigma, \eta) g(\eta) d\eta > 0. \tag{53}$$

By (12),

$$\int_{\underline{\eta}}^{\overline{\eta}} (1 - H(\phi_{NL}^*(\sigma, \eta))) C_{NL}(\sigma, \eta) g(\eta) d\eta > 0 \tag{54}$$

holds for $\sigma = \sigma_{NL}^*$. Given $qr < 1$ then $\lambda \simeq 1$ implies $\lambda > qr$ and (53) holds. We have then shown that there exists $\widehat{\lambda} < 1$ and $\widehat{\gamma} > 0$ such that if $(\gamma, \lambda) \in [0, \widehat{\gamma}] \times \left[\widehat{\lambda}, 1\right]$ then (51) holds, generically, in a small neighborhood of $\sigma = \sigma_{NL}^*$.

For when there is no leniency program, σ_{NL}^* is defined by:

$$\sigma_{NL}^* = qrp\left(qr \int_{\underline{\eta}}^{\overline{\eta}} C_{NL}(\sigma_{NL}^*, \eta) g(\eta) d\eta \right).$$

As it is the maximal fixed point then:

$$\sigma - qrp\left(qr \int_{\underline{\eta}}^{\overline{\eta}} C_{NL}(\sigma, \eta) g(\eta) d\eta \right) \gtreqqless 0 \text{ as } \sigma \gtreqqless \sigma_{NL}^*. \tag{55}$$

Hence, using (51), it follows from (55) that there exists $\widehat{\lambda} < 1$ and $\widehat{\gamma} > 0$ such that if $(\gamma, \lambda) \in [0, \widehat{\gamma}] \times \left[\widehat{\lambda}, 1\right]$ then $\exists \varepsilon > 0$ such that

$$\sigma - qrp\left(\lambda \int_{\underline{\eta}}^{\overline{\eta}} (1 - H(\phi_L^*(\sigma, \eta))) C_L(\sigma, \eta) g(\eta) d\eta \right. \tag{56}$$

$$\left. + qr \int_{\underline{\eta}}^{\overline{\eta}} H(\phi_L^*(\sigma, \eta)) C_L(\sigma, \eta) g(\eta) d\eta \right)$$

$$> \quad 0, \forall \sigma \geq \sigma_{NL}^* - \varepsilon.$$

Given the continuity of

$$p\left(\lambda \int_{\underline{\eta}}^{\overline{\eta}} (1 - H(\phi_L^*(\sigma, \eta))) C_L(\sigma, \eta) g(\eta) d\eta + qr \int_{\underline{\eta}}^{\overline{\eta}} H(\phi_L^*(\sigma, \eta)) C_L(\sigma, \eta) g(\eta) d\eta \right)$$

in σ (see the proof of Theorem 5), (56) implies the maximal fixed point σ_L^* is less than $\sigma_{NL}^* - \varepsilon$. Given (50) and having just shown

$$\lim_{\gamma \to 0} \sigma_{NL}^* > \lim_{\gamma \to 0} \sigma_L^*,$$

it follows that

$$\lim_{\gamma \to 0} C_L(\sigma_L^*) > \lim_{\gamma \to 0} C_{NL}(\sigma_{NL}^*).$$

■

Proof of Theorem 11. If $C(\sigma) > 0$ then $\widehat{\eta}(\sigma) > \underline{\eta}$ and $Y^*(\sigma, \eta) > \alpha\mu \ \forall \eta \in (1, \widehat{\eta}(\sigma)]$. Furthermore, since $Y^*(\sigma, \widehat{\eta}(\sigma)) > \alpha\mu$ and $Y^*(\sigma, \eta)$ is non-increasing in η (Lemma 2) then

$$\lim_{\eta \to 1} Y^*(\sigma, \eta) > \alpha\mu.$$

Recall

$$
\begin{aligned}
\phi(Y, \sigma, \eta) &= \frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu) - [1 - \delta(1-\kappa)][\sigma - \min\{\sigma, \theta\}]\gamma(Y-\alpha\mu)}{(\eta-1)[1-\delta(1-\kappa)]} \\
&= \frac{\{\delta(1-\sigma)(1-\kappa) - [1-\delta(1-\kappa)][\sigma - \min\{\sigma,\theta\}]\gamma\}(Y-\alpha\mu)}{(\eta-1)[1-\delta(1-\kappa)]}
\end{aligned}
$$

and

$$\phi^*(\sigma, \eta) \equiv \max\{\min\{\phi(Y^*(\sigma, \eta), \sigma, \eta), \overline{\pi}\}, \underline{\pi}\}$$

where this encompasses both the case of a full leniency program ($\theta = 0$) and no leniency program ($\theta = 1$). Given that $Y^*(\sigma, \eta)$ is bounded above $\alpha\mu$ as $\eta \to 1$ then

$$
\begin{aligned}
\lim_{\eta \to 1} \phi(Y^*(\sigma, \eta), \sigma, \eta) &= \lim_{\eta \to 1} \frac{\{\delta(1-\sigma)(1-\kappa) - [1-\delta(1-\kappa)][\sigma - \min\{\sigma, \theta\}]\gamma\}(Y^*(\sigma, \eta) - \alpha\mu)}{(\eta-1)[1-\delta(1-\kappa)]} \\
&= +\infty
\end{aligned}
$$

and, therefore,

$$\lim_{\eta \to 1} H(\phi^*(\sigma, \eta)) = \lim_{\eta \to 1} H(\max\{\min\{\phi(Y^*(\sigma, \eta), \sigma, \eta), \overline{\pi}\}, \underline{\pi}\}) = 1.$$

Thus, when η is close to one, if a stable cartel forms (that is, $\phi^*(\sigma, \eta) < \overline{\pi}$) then it is fully stable (that is, $\phi^*(\sigma, \eta) = \overline{\pi}$).

Next note that

$$C(\sigma, \eta) = \frac{\kappa(1-\sigma)H(\phi^*(\sigma, \eta))}{1 - (1-\kappa)(1-\sigma)H(\phi^*(\sigma, \eta))}$$

and, therefore,

$$\lim_{\eta \to 1} C(\sigma, \eta) = \lim_{\eta \to 1} \frac{\kappa(1-\sigma)H(\phi^*(\sigma, \eta))}{1 - (1-\kappa)(1-\sigma)H(\phi^*(\sigma, \eta))} = \frac{\kappa(1-\sigma)}{1 - (1-\kappa)(1-\sigma)}.$$

We then have:

$$
\begin{aligned}
&\lim_{\eta \to 1}[C_L(\sigma_L^*, \eta) - C_{NL}(\sigma_{NL}^*, \eta)] \\
&= \frac{\kappa(1-\sigma_L^*)}{1 - (1-\kappa)(1-\sigma_L^*)} - \frac{\kappa(1-\sigma_{NL}^*)}{1 - (1-\kappa)(1-\sigma_{NL}^*)} \\
&= \kappa\left[\frac{(1-\sigma_L^*)[1-(1-\kappa)(1-\sigma_{NL}^*)] - (1-\sigma_{NL}^*)[1-(1-\kappa)(1-\sigma_L^*)]}{[1-(1-\kappa)(1-\sigma_L^*)][1-(1-\kappa)(1-\sigma_{NL}^*)]}\right] \\
&= \frac{\kappa(\sigma_{NL}^* - \sigma_L^*)}{[1-(1-\kappa)(1-\sigma_L^*)][1-(1-\kappa)(1-\sigma_{NL}^*)]}.
\end{aligned}
$$

■

Proof of Theorem 12. Let us first show: if $\sigma \in (0, \omega)$ and $\widehat{\eta}_{NL}(\sigma) \in \left(\underline{\eta}, \overline{\eta}\right)$ then $\widehat{\eta}_L(\sigma) < \widehat{\eta}_{NL}(\sigma)$. By the definition of $\widehat{\eta}$, if $\widehat{\eta} \in \left(\underline{\eta}, \overline{\eta}\right)$ then there is a (maximal) fixed point in Y of $\psi(Y, \sigma, \eta)$ such that $Y > \alpha\mu$ for $\eta = \widehat{\eta}$ but not for $\eta > \widehat{\eta}$:

$$\exists Y^*(\sigma, \widehat{\eta}) \in (\alpha\mu, \mu] \text{ such that } Y \lessgtr \psi(Y, \sigma, \widehat{\eta}) \text{ as } Y \gtrless Y^*(\sigma, \widehat{\eta}) \qquad (57)$$
$$\nexists Y \in (\alpha\mu, \mu] \text{ such that } Y = \psi(Y, \sigma, \eta), \forall \eta \in (\widehat{\eta}, \overline{\eta}].$$

Recall that

$$\psi(Y, \sigma, \eta) = \begin{cases} \int_{\underline{\pi}}^{\phi(Y, \sigma, \eta)} \{(1 - \delta)\pi + \delta[(1 - \sigma)Y + \sigma W] - (1 - \delta)\sigma\gamma(Y - \alpha\mu)\} h(\pi) d\pi & \text{if } \sigma \le \theta \\ + \int_{\phi(Y, \sigma, \eta)}^{\overline{\pi}} [(1 - \delta)\alpha\pi + \delta W - (1 - \delta)\sigma\gamma(Y - \alpha\mu)] h(\pi) d\pi & \\ \\ \int_{\underline{\pi}}^{\phi(Y, \sigma, \eta)} \{(1 - \delta)\pi + \delta[(1 - \sigma)Y + \sigma W] - (1 - \delta)\sigma\gamma(Y - \alpha\mu)\} h(\pi) d\pi & \text{if } \theta < \sigma \\ + \int_{\phi(Y, \sigma, \eta)}^{\overline{\pi}} [(1 - \delta)\alpha\pi + \delta W - (1 - \delta)\omega\gamma(Y - \alpha\mu)] h(\pi) d\pi & \end{cases}.$$

and

$$\phi(Y, \sigma, \eta) = \frac{\delta(1 - \sigma)(1 - \kappa)(Y - \alpha\mu) - [1 - \delta(1 - \kappa)][\sigma - \min\{\sigma, \theta\}]\gamma(Y - \alpha\mu)}{(\eta - 1)[1 - \delta(1 - \kappa)]}.$$

Let us next argue that

$$\phi(Y^*(\sigma, \widehat{\eta}), \sigma, \widehat{\eta}) \in (\underline{\pi}, \overline{\pi}]. \qquad (58)$$

Obviously, $Y^*(\sigma, \widehat{\eta}) > \alpha\mu$ implies $\phi(Y^*(\sigma, \widehat{\eta}), \sigma, \eta) > \underline{\pi}$. If $\phi(Y^*(\sigma, \widehat{\eta}), \sigma, \eta) > \overline{\pi}$ then, by the continuity of $\phi(Y, \sigma, \eta)$ in η, it follows that $\exists\varepsilon > 0$ such that $\phi(Y^*(\sigma, \widehat{\eta}), \sigma, \eta) > \overline{\pi} \ \forall \eta \in (\widehat{\eta}, \widehat{\eta} + \varepsilon)$. Given that η affects $\psi(Y, \sigma, \eta)$ only through $\phi(Y, \sigma, \eta)$ - and recalling that $H(\overline{\pi}) = 1$ - then

$$\psi(Y^*(\sigma, \widehat{\eta}), \sigma, \widehat{\eta}) = \psi(Y^*(\sigma, \widehat{\eta}), \sigma, \eta) \forall \eta \in (\widehat{\eta}, \widehat{\eta} + \varepsilon)$$

which implies $Y^*(\sigma, \widehat{\eta})$ is a fixed point to $\psi(Y, \sigma, \eta) \ \forall \eta \in (\widehat{\eta}, \widehat{\eta} + \varepsilon)$ which contradicts (57). We then conclude $\phi(Y^*(\sigma, \widehat{\eta}), \sigma, \eta) \le \overline{\pi}$ and (58) is true.

Using (58) for when there is no leniency program, $\widehat{\eta}_{NL}(\sigma) \in \left(\underline{\eta}, \overline{\eta}\right)$ implies $\phi_{NL}(Y^*_{NL}(\sigma, \widehat{\eta}_{NL}), \sigma, \widehat{\eta}_{NL}) \in (\underline{\pi}, \overline{\pi}]$. Since ϕ is increasing in Y, it then follows:

$$\overline{\pi} > \phi_{NL}(Y, \sigma, \widehat{\eta}_{NL}), \forall Y \in [\alpha\mu, Y^*_{NL}(\sigma, \widehat{\eta}_{NL})). \qquad (59)$$

In the proof of Theorem 6 it was shown: if $\sigma > 0$ then $\phi_{NL}(Y, \sigma, \eta) > \phi_L(Y, \sigma, \eta)$. Given it is assumed $\sigma > 0$, (59) then implies

$$\overline{\pi} \ge \phi_{NL}(Y, \sigma, \widehat{\eta}_{NL}) > \phi_L(Y, \sigma, \widehat{\eta}_{NL}), \forall Y \in [\alpha\mu, Y^*_{NL}(\sigma, \widehat{\eta}_{NL})] \qquad (60)$$

Now consider:

$$\psi_{NL}(Y,\sigma,\eta) - \psi_L(Y,\sigma,\eta)$$

$$= \int_{\underline{\pi}}^{\phi_{NL}(Y,\sigma,\eta)} \{(1-\delta)\pi + \delta[(1-\sigma)Y + \sigma W] - (1-\delta)\sigma\gamma(Y-\alpha\mu)\} h(\pi) d\pi$$

$$+ \int_{\phi_{NL}(Y,\sigma,\eta)}^{\overline{\pi}} [(1-\delta)\alpha\pi + \delta W - (1-\delta)\sigma\gamma(Y-\alpha\mu)] h(\pi) d\pi$$

$$- \int_{\underline{\pi}}^{\phi_L(Y,\sigma,\eta)} \{(1-\delta)\pi + \delta[(1-\sigma)Y + \sigma W] - (1-\delta)\sigma\gamma(Y-\alpha\mu)\} h(\pi) d\pi$$

$$- \int_{\phi_L(Y,\sigma,\eta)}^{\overline{\pi}} [(1-\delta)\alpha\pi + \delta W - (1-\delta)\omega\gamma(Y-\alpha\mu)] h(\pi) d\pi$$

After some simplifying steps:

$$\psi_{NL}(Y,\sigma,\eta) - \psi_L(Y,\sigma,\eta)$$

$$= \int_{\phi_L(Y,\sigma,\eta)}^{\phi_{NL}(Y,\sigma,\eta)} \{(1-\delta)(1-\alpha)\pi + \delta(1-\sigma)(Y-W) - (1-\delta)(\sigma-\omega)\gamma(Y-\alpha\mu)\} h(\pi) d\pi$$

$$+ \int_{\phi_{NL}(Y,\sigma,\eta)}^{\overline{\pi}} [(1-\delta)\alpha\pi + \delta W - (1-\delta)(\sigma-\omega)\gamma(Y-\alpha\mu)] h(\pi) d\pi.$$

Given $\sigma \in (0,\omega)$ and using (60), we have:

$$\psi_{NL}(Y,\sigma,\widehat{\eta}_{NL}) - \psi_L(Y,\sigma,\widehat{\eta}_{NL}) > 0. \qquad (61)$$

Next note that it follows from $\widehat{\eta}_{NL} \in (\underline{\eta},\overline{\eta})$ that:

$$\psi_{NL}(Y,\sigma,\widehat{\eta}_{NL}) \leq Y \;\forall Y \in [\alpha\mu, Y^*_{NL}(\sigma,\widehat{\eta}_{NL})]$$
$$\psi_{NL}(Y,\sigma,\widehat{\eta}_{NL}) < Y \;\forall Y \in (Y^*_{NL}(\sigma,\widehat{\eta}_{NL}),\mu].$$

Using (61), this implies

$$\psi_L(Y,\sigma,\widehat{\eta}_{NL}) < Y \;\forall Y \in [\alpha\mu, Y^*(\sigma,\widehat{\eta}_{NL})]$$
$$\psi_L(Y,\sigma,\widehat{\eta}_{NL}) < Y \;\forall Y \in (Y^*(\sigma,\widehat{\eta}_{NL}),\mu],$$

and, therefore, $\widehat{\eta}_L(\sigma) < \widehat{\eta}_{NL}(\sigma)$.

We have thus far shown: $\widehat{\eta}_L(\sigma^*_{NL}) < \widehat{\eta}_{NL}(\sigma^*_{NL})$. By Lemma 3, $\widehat{\eta}(\sigma)$ is non-increasing in σ. Hence, if $\sigma^*_L \geq \sigma^*_{NL}$ then $\widehat{\eta}_L(\sigma^*_L) \leq \widehat{\eta}_L(\sigma^*_{NL})$ which implies $\widehat{\eta}_L(\sigma^*_L) < \widehat{\eta}_{NL}(\sigma^*_{NL})$. $\blacksquare$

8 Appendix B: Additional Material on the Proof of Theorem 7

8.1 Assume $\sigma < \theta$

- Assume $\phi(Y) = \underline{\pi}$

$$\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{(\eta-1)[1-\delta(1-\kappa)]} \leq \underline{\pi} \Rightarrow Y \leq \alpha\mu + \frac{\underline{\pi}(\eta-1)[1-\delta(1-\kappa)]}{\delta(1-\sigma)(1-\kappa)} \equiv A$$

54

$$\psi\left(Y\right) \;=\; \int_{\underline{\pi}}^{\overline{\pi}}\left[\left(1-\delta\right)\alpha\pi+\delta W-\left(1-\delta\right)\sigma\gamma\left(Y-\alpha\mu\right)\right]h\left(\pi\right)d\pi$$

$$=\; \int_{\underline{\pi}}^{\overline{\pi}}\left[\left(1-\delta\right)\alpha\pi+\delta\frac{\left(1-\kappa\right)\left(1-\delta\right)\alpha\mu+\kappa Y}{1-\delta\left(1-\kappa\right)}-\left(1-\delta\right)\sigma\gamma\left(Y-\alpha\mu\right)\right]h\left(\pi\right)d\pi$$

$$=\; \left(1-\delta\right)\alpha\mu+\delta\frac{\left(1-\kappa\right)\left(1-\delta\right)\alpha\mu+\kappa Y}{1-\delta\left(1-\kappa\right)}-\left(1-\delta\right)\sigma\gamma\left(Y-\alpha\mu\right)$$

$$\psi'\left(Y\right)=\frac{\delta\kappa}{1-\delta\left(1-\kappa\right)}-\left(1-\delta\right)\sigma\gamma\in\left(0,1\right)\ \text{if}\ \gamma\simeq0.$$

- Assume $\phi\left(Y\right)\in\left(\underline{\pi},\overline{\pi}\right)$

$$\psi\left(Y\right) \;=\; \int_{\underline{\pi}}^{\phi\left(Y\right)}\left[\left(1-\delta\right)\pi+\delta\left(1-\sigma\right)Y+\delta\sigma W-\left(1-\delta\right)\sigma\gamma\left(Y-\alpha\mu\right)\right]h\left(\pi\right)d\pi$$

$$+\int_{\phi\left(Y\right)}^{\overline{\pi}}\left[\left(1-\delta\right)\alpha\pi+\delta W-\left(1-\delta\right)\sigma\gamma\left(Y-\alpha\mu\right)\right]h\left(\pi\right)d\pi$$

$$\psi'\left(Y\right) \;=\; \left[\left(1-\delta\right)\phi\left(Y\right)+\delta\left(1-\sigma\right)Y+\delta\sigma W-\left(1-\delta\right)\sigma\gamma\left(Y-\alpha\mu\right)\right]h\left(\phi\left(Y\right)\right)\phi'\left(Y\right)$$

$$+\int_{\underline{\pi}}^{\phi\left(Y\right)}\left[\delta\left(1-\sigma\right)+\delta\sigma\frac{\partial W}{\partial Y}-\left(1-\delta\right)\sigma\gamma\right]h\left(\pi\right)d\pi$$

$$-\left[\left(1-\delta\right)\alpha\phi\left(Y\right)+\delta W-\left(1-\delta\right)\sigma\gamma\left(Y-\alpha\mu\right)\right]h\left(\phi\left(Y\right)\right)\phi'\left(Y\right)$$

$$+\int_{\phi\left(Y\right)}^{\overline{\pi}}\left[\delta\frac{\partial W}{\partial Y}-\left(1-\delta\right)\sigma\gamma\right]h\left(\pi\right)d\pi$$

$$\psi'\left(Y\right) \;=\; \left[\left(1-\delta\right)\phi\left(Y\right)+\delta\left(1-\sigma\right)Y+\delta\sigma W-\left(1-\delta\right)\sigma\gamma\left(Y-\alpha\mu\right)\right]h\left(\phi\left(Y\right)\right)\phi'\left(Y\right)$$

$$+\left[\delta\left(1-\sigma\right)+\delta\sigma\frac{\partial W}{\partial Y}-\left(1-\delta\right)\sigma\gamma\right]H\left(\phi\left(Y\right)\right)$$

$$+\left[\delta\frac{\partial W}{\partial Y}-\left(1-\delta\right)\sigma\gamma\right]\left[1-H\left(\phi\left(Y\right)\right)\right]$$

$$-\left[\left(1-\delta\right)\alpha\phi\left(Y\right)+\delta W-\left(1-\delta\right)\sigma\gamma\left(Y-\alpha\mu\right)\right]h\left(\phi\left(Y\right)\right)\phi'\left(Y\right)$$

$$\psi'\left(Y\right) \;=\; \left[\left(1-\delta\right)\phi\left(Y\right)+\delta\left(1-\sigma\right)Y+\delta\sigma W-\left(1-\delta\right)\sigma\gamma\left(Y-\alpha\mu\right)\right]h\left(\phi\left(Y\right)\right)\phi'\left(Y\right)$$

$$+\left[\delta\left(1-\sigma\right)+\delta\sigma\frac{\partial W}{\partial Y}\right]H\left(\phi\left(Y\right)\right)+\delta\frac{\partial W}{\partial Y}\left[1-H\left(\phi\left(Y\right)\right)\right]-\left(1-\delta\right)\sigma\gamma$$

$$-\left[\left(1-\delta\right)\alpha\phi\left(Y\right)+\delta W-\left(1-\delta\right)\sigma\gamma\left(Y-\alpha\mu\right)\right]h\left(\phi\left(Y\right)\right)\phi'\left(Y\right)\gamma$$

$$\psi'\left(Y\right) \;=\; \left[\left(1-\delta\right)\left(1-\alpha\right)\phi\left(Y\right)+\delta\left(1-\sigma\right)\left(Y-W\right)\right]h\left(\phi\left(Y\right)\right)\phi'\left(Y\right)-\left(1-\delta\right)\sigma\gamma$$

$$+\left[\delta\left(1-\sigma\right)+\delta\sigma\frac{\partial W}{\partial Y}\right]H\left(\phi\left(Y\right)\right)+\delta\frac{\partial W}{\partial Y}\left[1-H\left(\phi\left(Y\right)\right)\right]$$

$$\phi'(Y) = \frac{\delta(1-\sigma)(1-\kappa)}{(\eta-1)[1-\delta(1-\kappa)]}$$

$$
\begin{aligned}
\psi''(Y) &= \left[(1-\delta)(1-\alpha)\phi'(Y) + \delta(1-\sigma)\left(1 - \frac{\partial W}{\partial Y}\right)\right] h(\phi(Y))\phi'(Y) \\
&\quad + [(1-\delta)(1-\alpha)\phi(Y) + \delta(1-\sigma)(Y-W)] h'(\phi(Y))(\phi'(Y))^2 \\
&\quad + \left[\delta(1-\sigma) + \delta\sigma\frac{\partial W}{\partial Y}\right] h(\phi(Y))\phi'(Y) - \delta\frac{\partial W}{\partial Y} h(\phi(Y))\phi'(Y)
\end{aligned}
$$

$$
\begin{aligned}
\psi''(Y) &= \left[(1-\delta)(1-\alpha)\phi'(Y) + \delta(1-\sigma)\left(1 - \frac{\partial W}{\partial Y}\right)\right] h(\phi(Y))\phi'(Y) \\
&\quad + [(1-\delta)(1-\alpha)\phi(Y) + \delta(1-\sigma)(Y-W)] h'(\phi(Y))(\phi'(Y))^2 \\
&\quad + \delta(1-\sigma)\left(1 - \frac{\partial W}{\partial Y}\right) h(\phi(Y))\phi'(Y)
\end{aligned}
$$

$$
\begin{aligned}
\psi''(Y) &= \left[(1-\delta)(1-\alpha)\phi'(Y) + 2\delta(1-\sigma)\left(1 - \frac{\partial W}{\partial Y}\right)\right] h(\phi(Y))\phi'(Y) \\
&\quad + [(1-\delta)(1-\alpha)\phi(Y) + \delta(1-\sigma)(Y-W)] h'(\phi(Y))(\phi'(Y))^2
\end{aligned}
$$

$$
\begin{aligned}
\psi'''(Y) &= \left[(1-\delta)(1-\alpha)\phi'(Y) + 2\delta(1-\sigma)\left(1 - \frac{\partial W}{\partial Y}\right)\right] h'(\phi(Y))(\phi'(Y))^2 \\
&\quad + \left[(1-\delta)(1-\alpha)\phi'(Y) + \delta(1-\sigma)\left(1 - \frac{\partial W}{\partial Y}\right)\right] h'(\phi(Y))(\phi'(Y))^2 \\
&\quad + [(1-\delta)(1-\alpha)\phi(Y) + \delta(1-\sigma)(Y-W)] h''(\phi(Y))(\phi'(Y))^3
\end{aligned}
$$

If H is uniform then

$$\psi''(Y) = \left[(1-\delta)(1-\alpha)\phi'(Y) + 2\delta(1-\sigma)\left(1 - \frac{\partial W}{\partial Y}\right)\right]\left(\frac{\phi'(Y)}{\overline{\pi} - \underline{\pi}}\right)$$

$$\psi'''(Y) = 0$$

- Assume $\phi(Y) = \overline{\pi}$

$$\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu)}{(\eta-1)[1-\delta(1-\kappa)]} \geq \overline{\pi} \Rightarrow Y \geq \alpha\mu + \frac{\overline{\pi}(\eta-1)[1-\delta(1-\kappa)]}{\delta(1-\sigma)(1-\kappa)} \equiv B$$

$$
\begin{aligned}
\psi(Y) &= \int_{\underline{\pi}}^{\overline{\pi}} [(1-\delta)\pi + \delta Y - \delta\sigma(Y-W) - (1-\delta)\sigma\gamma(Y-\alpha\mu)] h(\pi) d\pi \\
&= (1-\delta)\mu + \delta Y - \delta\sigma(Y-W) - (1-\delta)\sigma\gamma(Y-\alpha\mu)
\end{aligned}
$$

$$\psi'(Y) = \delta - \delta\sigma\left(1 - \frac{\partial W}{\partial Y}\right) - (1-\delta)\sigma\gamma$$

$$= \delta - \delta\sigma\left(\frac{(1-\kappa)(1-\delta)}{1-\delta(1-\kappa)}\right) - (1-\delta)\sigma\gamma$$

$$= \delta\left(\frac{1-\delta(1-\kappa)-\sigma(1-\kappa)(1-\delta)}{1-\delta(1-\kappa)}\right) - (1-\delta)\sigma\gamma \in (0,1) \text{ if } \gamma \simeq 0.$$

- Solve $\psi(B(\eta),\eta) = B(\eta)$ for η

$$(\psi(B,\eta) =)(1-\delta)\mu + \delta B - \delta\sigma\left(\frac{(1-\kappa)(1-\delta)(B-\alpha\mu)}{1-\delta(1-\kappa)}\right) - (1-\delta)\sigma\gamma(B-\alpha\mu) = B$$

and substituting for B:

$$\begin{aligned}
& (1-\delta)\mu + \delta\left(\alpha\mu + \frac{\overline{\pi}(\widehat{\eta}-1)[1-\delta(1-\kappa)]}{\delta(1-\sigma)(1-\kappa)}\right) \\
& - \left[\delta\sigma\left(\frac{(1-\kappa)(1-\delta)}{1-\delta(1-\kappa)}\right) + (1-\delta)\sigma\gamma\right]\left(\frac{\overline{\pi}(\widehat{\eta}-1)[1-\delta(1-\kappa)]}{\delta(1-\sigma)(1-\kappa)}\right) \\
= & \ \alpha\mu + \frac{\overline{\pi}(\widehat{\eta}-1)[1-\delta(1-\kappa)]}{\delta(1-\sigma)(1-\kappa)}
\end{aligned}$$

$$\begin{aligned}
& (1-\delta)(1-\alpha)\mu + \delta\left(\frac{\overline{\pi}(\widehat{\eta}-1)[1-\delta(1-\kappa)]}{\delta(1-\sigma)(1-\kappa)}\right) \\
& - \left[\delta\sigma\left(\frac{(1-\kappa)(1-\delta)}{1-\delta(1-\kappa)}\right) + (1-\delta)\sigma\gamma\right]\left(\frac{\overline{\pi}(\widehat{\eta}-1)[1-\delta(1-\kappa)]}{\delta(1-\sigma)(1-\kappa)}\right) \\
= & \ \frac{\overline{\pi}(\widehat{\eta}-1)[1-\delta(1-\kappa)]}{\delta(1-\sigma)(1-\kappa)}
\end{aligned}$$

$$(1-\delta)(1-\alpha)\mu = \left(\frac{\overline{\pi}(\widehat{\eta}-1)[1-\delta(1-\kappa)]}{\delta(1-\sigma)(1-\kappa)}\right)\left[1 - \delta + \delta\sigma\left(\frac{(1-\kappa)(1-\delta)}{1-\delta(1-\kappa)}\right) + (1-\delta)\sigma\gamma\right]$$

$$\begin{aligned}
(1-\delta)(1-\alpha)\mu = & \left(\frac{\overline{\pi}(\widehat{\eta}-1)[1-\delta(1-\kappa)]}{\delta(1-\sigma)(1-\kappa)}\right) \times \\
& \left[\frac{(1-\delta)(1-\delta(1-\kappa)) + \delta\sigma(1-\kappa)(1-\delta) + (1-\delta)\sigma\gamma(1-\delta(1-\kappa))}{1-\delta(1-\kappa)}\right]
\end{aligned}$$

$$\overline{\pi}(\widehat{\eta}-1)\left[\frac{(1-\delta(1-\kappa)) + \delta\sigma(1-\kappa) + \sigma\gamma(1-\delta(1-\kappa))}{\delta(1-\sigma)(1-\kappa)}\right] = (1-\alpha)\mu$$

$$\widehat{\eta} = 1 + \left(\frac{\mu}{\overline{\pi}}\right)\left(\frac{\delta(1-\sigma)(1-\kappa)(1-\alpha)}{\delta\sigma(1-\kappa) + (1+\sigma\gamma)(1-\delta(1-\kappa))}\right)$$

8.2 Assume $\sigma > \theta$

- Assume $\phi(Y) = \underline{\pi}$.

$$\frac{\delta(1-\sigma)(1-\kappa)(Y-\alpha\mu) - [1 - \delta(1-\kappa)](\sigma - \theta)\gamma(Y-\alpha\mu)}{(\eta - 1)[1 - \delta(1-\kappa)]} \leq \underline{\pi}$$

$$Y \leq \alpha\mu + \frac{\underline{\pi}(\eta - 1)[1 - \delta(1-\kappa)]}{\delta(1-\sigma)(1-\kappa) - [1 - \delta(1-\kappa)](\sigma - \theta)\gamma} \equiv C$$

$$\begin{aligned}
\psi(Y) &= \int_{\underline{\pi}}^{\overline{\pi}} [(1-\delta)\alpha\pi + \delta W - (1-\delta)\omega\gamma(Y-\alpha\mu)] h(\pi) d\pi \\
&= (1-\delta)\alpha\mu + \delta\frac{(1-\kappa)(1-\delta)\alpha\mu + \kappa Y}{1 - \delta(1-\kappa)} - (1-\delta)\omega\gamma(Y-\alpha\mu)
\end{aligned}$$

$$\psi'(Y) = \frac{\delta\kappa}{1 - \delta(1-\kappa)} - (1-\delta)\omega\gamma \in (0,1) \;\; \text{if } \gamma \simeq 0.$$

- Assume $\phi(Y) \in (\underline{\pi}, \overline{\pi})$.

$$\begin{aligned}
\psi(Y) &= \int_{\underline{\pi}}^{\phi(Y)} [(1-\delta)\pi + \delta(1-\sigma)Y + \delta\sigma W - (1-\delta)\sigma\gamma(Y-\alpha\mu)] h(\pi) d\pi \\
&\quad + \int_{\phi(Y)}^{\overline{\pi}} [(1-\delta)\alpha\pi + \delta W - (1-\delta)\omega\gamma(Y-\alpha\mu)] h(\pi) d\pi
\end{aligned}$$

$$\begin{aligned}
\psi'(Y) &= [(1-\delta)\phi(Y) + \delta(1-\sigma)Y + \delta\sigma W - (1-\delta)\sigma\gamma(Y-\alpha\mu)] h(\phi(Y))\phi'(Y) \\
&\quad + \int_{\underline{\pi}}^{\phi(Y)} \left[\delta(1-\sigma) + \delta\sigma\frac{\partial W}{\partial Y} - (1-\delta)\sigma\gamma\right] h(\pi) d\pi \\
&\quad - [(1-\delta)\alpha\phi(Y) + \delta W - (1-\delta)\omega\gamma(Y-\alpha\mu)] h(\phi(Y))\phi'(Y) \\
&\quad + \int_{\phi(Y)}^{\overline{\pi}} \left[\delta\frac{\partial W}{\partial Y} - (1-\delta)\omega\gamma\right] h(\pi) d\pi
\end{aligned}$$

$$\begin{aligned}
\psi'(Y) &= [(1-\delta)(1-\alpha)\phi(Y) + \delta(1-\sigma)(Y-W) - (1-\delta)(\sigma - \omega)\gamma(Y-\alpha\mu)] \times \\
&\quad h(\phi(Y))\phi'(Y) + \left[\delta(1-\sigma) + \delta\sigma\frac{\partial W}{\partial Y} - (1-\delta)\sigma\gamma\right] H(\phi(Y)) \\
&\quad + \left[\delta\frac{\partial W}{\partial Y} - (1-\delta)\omega\gamma\right](1 - H(\phi(Y)))
\end{aligned}$$

$$\psi''\left(Y\right) = \left[\left(1-\delta\right)\left(1-\alpha\right)\phi'\left(Y\right)+\delta\left(1-\sigma\right)\left(1-\frac{\partial W}{\partial Y}\right)-\left(1-\delta\right)\left(\sigma-\omega\right)\gamma\right]\times$$
$$h\left(\phi\left(Y\right)\right)\phi'\left(Y\right)$$
$$+\left[\left(1-\delta\right)\left(1-\alpha\right)\phi\left(Y\right)+\delta\left(1-\sigma\right)\left(Y-W\right)-\left(1-\delta\right)\left(\sigma-\omega\right)\gamma\left(Y-\alpha\mu\right)\right]\times$$
$$h'\left(\phi\left(Y\right)\right)\left(\phi'\left(Y\right)\right)^{2}$$
$$+\left[\delta\left(1-\sigma\right)+\delta\sigma\frac{\partial W}{\partial Y}-\left(1-\delta\right)\sigma\gamma\right]h\left(\phi\left(Y\right)\right)\phi'\left(Y\right)$$
$$-\left[\delta\frac{\partial W}{\partial Y}-\left(1-\delta\right)\omega\gamma\right]h\left(\phi\left(Y\right)\right)\phi'\left(Y\right)$$

$$\psi''\left(Y\right) = \left[\left(1-\delta\right)\left(1-\alpha\right)\phi'\left(Y\right)+2\delta\left(1-\sigma\right)\left(1-\frac{\partial W}{\partial Y}\right)-2\left(1-\delta\right)\left(\sigma-\omega\right)\gamma\right]\times$$
$$h\left(\phi\left(Y\right)\right)\phi'\left(Y\right)$$
$$+\left[\left(1-\delta\right)\left(1-\alpha\right)\phi\left(Y\right)+\delta\left(1-\sigma\right)\left(Y-W\right)-\left(1-\delta\right)\left(\sigma-\omega\right)\gamma\left(Y-\alpha\mu\right)\right]\times$$
$$h'\left(\phi\left(Y\right)\right)\left(\phi'\left(Y\right)\right)^{2}$$
$$\phi'\left(Y\right)=\frac{\delta\left(1-\sigma\right)\left(1-\kappa\right)-\left[1-\delta\left(1-\kappa\right)\right]\left(\sigma-\theta\right)\gamma}{\left(\eta-1\right)\left[1-\delta\left(1-\kappa\right)\right]}$$

If H is uniform then
$$\psi''\left(Y\right) = \left[\left(1-\delta\right)\left(1-\alpha\right)\phi'\left(Y\right)+2\delta\left(1-\sigma\right)\left(1-\frac{\partial W}{\partial Y}\right)-2\left(1-\delta\right)\left(\sigma-\omega\right)\gamma\right]\times$$
$$\left(\frac{\phi'\left(Y\right)}{\overline{\pi}-\underline{\pi}}\right)$$
$$\psi'''\left(Y\right)=0$$

$-$ Assume $\phi\left(Y\right)=\overline{\pi}$.
$$\frac{\delta\left(1-\sigma\right)\left(1-\kappa\right)\left(Y-\alpha\mu\right)-\left[1-\delta\left(1-\kappa\right)\right]\left(\sigma-\theta\right)\gamma\left(Y-\alpha\mu\right)}{\left(\eta-1\right)\left[1-\delta\left(1-\kappa\right)\right]}\geq\overline{\pi}$$
$$\left[\delta\left(1-\sigma\right)\left(1-\kappa\right)-\left(1-\delta\left(1-\kappa\right)\right)\left(\sigma-\theta\right)\gamma\right]\left(Y-\alpha\mu\right)\geq\overline{\pi}\left(\eta-1\right)\left[1-\delta\left(1-\kappa\right)\right]$$
$$Y\geq\alpha\mu+\frac{\overline{\pi}\left(\eta-1\right)\left[1-\delta\left(1-\kappa\right)\right]}{\delta\left(1-\sigma\right)\left(1-\kappa\right)-\left[1-\delta\left(1-\kappa\right)\right]\left(\sigma-\theta\right)\gamma}\equiv D$$

$$\psi\left(Y\right) = \int_{\underline{\pi}}^{\overline{\pi}}\left[\left(1-\delta\right)\pi+\delta\left(1-\sigma\right)Y+\delta\sigma W-\left(1-\delta\right)\sigma\gamma\left(Y-\alpha\mu\right)\right]h\left(\pi\right)d\pi$$
$$= \left(1-\delta\right)\mu+\delta\left(1-\sigma\right)Y+\delta\sigma W-\left(1-\delta\right)\sigma\gamma\left(Y-\alpha\mu\right)$$
$$= \left(1-\delta\right)\mu+\delta Y-\delta\sigma\left(Y-W\right)-\left(1-\delta\right)\sigma\gamma\left(Y-\alpha\mu\right)$$

$$\psi'\left(Y\right) = \delta-\delta\sigma\left(\frac{\left(1-\kappa\right)\left(1-\delta\right)}{1-\delta\left(1-\kappa\right)}\right)-\left(1-\delta\right)\sigma\gamma$$
$$= \delta\left(1-\sigma\left(\frac{\left(1-\kappa\right)\left(1-\delta\right)}{1-\delta\left(1-\kappa\right)}\right)\right)-\left(1-\delta\right)\sigma\gamma<1\text{ if }\gamma\simeq0.$$

- Solve $\psi\left(D\left(\eta\right),\eta\right)=D\left(\eta\right)$ for η. First simplify $\psi\left(Y,\eta\right)=Y$,

$$\left(\psi\left(Y,\eta\right)=\right)\left(1-\delta\right)\mu+\delta Y-\delta\sigma\left(\frac{\left(1-\kappa\right)\left(1-\delta\right)\left(Y-\alpha\mu\right)}{1-\delta\left(1-\kappa\right)}\right)-\left(1-\delta\right)\sigma\gamma\left(Y-\alpha\mu\right)=Y$$

$$\left(1-\delta\right)\mu+\delta Y-\left(\frac{\left(1-\kappa\right)\left(1-\delta\right)\delta\sigma+\left[1-\delta\left(1-\kappa\right)\right]\left(1-\delta\right)\sigma\gamma}{1-\delta\left(1-\kappa\right)}\right)\left(Y-\alpha\mu\right) \;=\; Y$$

$$\left(1-\delta\right)\mu-\left(\frac{\left(1-\kappa\right)\left(1-\delta\right)\delta\sigma+\left[1-\delta\left(1-\kappa\right)\right]\left(1-\delta\right)\sigma\gamma}{1-\delta\left(1-\kappa\right)}\right)\left(Y-\alpha\mu\right) \;=\; Y\left(1-\delta\right)$$

$$\mu-\left(\frac{\left(1-\kappa\right)\delta\sigma+\left(1-\delta\left(1-\kappa\right)\right)\sigma\gamma}{1-\delta\left(1-\kappa\right)}\right)\left(Y-\alpha\mu\right) \;=\; Y$$

and then substitute for D,

$$\mu-\left(\frac{\left(1-\kappa\right)\delta\sigma+\left(1-\delta\left(1-\kappa\right)\right)\sigma\gamma}{1-\delta\left(1-\kappa\right)}\right)\left(\frac{\overline{\pi}\left(\eta-1\right)\left[1-\delta\left(1-\kappa\right)\right]}{\delta\left(1-\sigma\right)\left(1-\kappa\right)-\left[1-\delta\left(1-\kappa\right)\right]\left(\sigma-\theta\right)\gamma}\right)$$
$$=\;\alpha\mu+\frac{\overline{\pi}\left(\eta-1\right)\left[1-\delta\left(1-\kappa\right)\right]}{\delta\left(1-\sigma\right)\left(1-\kappa\right)-\left[1-\delta\left(1-\kappa\right)\right]\left(\sigma-\theta\right)\gamma}$$

$$\left(1-\alpha\right)\mu \;=\; \frac{\overline{\pi}\left(\eta-1\right)\left[1-\delta\left(1-\kappa\right)\right]}{\delta\left(1-\sigma\right)\left(1-\kappa\right)-\left[1-\delta\left(1-\kappa\right)\right]\left(\sigma-\theta\right)\gamma}$$
$$+\left(\frac{\left(1-\kappa\right)\delta\sigma+\left(1-\delta\left(1-\kappa\right)\right)\sigma\gamma}{1-\delta\left(1-\kappa\right)}\right)\times$$
$$\left(\frac{\overline{\pi}\left(\eta-1\right)\left[1-\delta\left(1-\kappa\right)\right]}{\delta\left(1-\sigma\right)\left(1-\kappa\right)-\left[1-\delta\left(1-\kappa\right)\right]\left(\sigma-\theta\right)\gamma}\right)$$

$$\left(1-\alpha\right)\mu \;=\; \left(\frac{\overline{\pi}\left(\eta-1\right)\left[1-\delta\left(1-\kappa\right)\right]}{\delta\left(1-\sigma\right)\left(1-\kappa\right)-\left[1-\delta\left(1-\kappa\right)\right]\left(\sigma-\theta\right)\gamma}\right)\times$$
$$\left(\frac{\left(1-\delta\left(1-\kappa\right)\right)+\left(1-\kappa\right)\delta\sigma+\left(1-\delta\left(1-\kappa\right)\right)\sigma\gamma}{1-\delta\left(1-\kappa\right)}\right)$$

$$\left(1-\alpha\right)\mu \;=\; \overline{\pi}\left(\eta-1\right)\left(\frac{\left(1-\delta\left(1-\kappa\right)\right)+\left(1-\kappa\right)\delta\sigma+\left(1-\delta\left(1-\kappa\right)\right)\sigma\gamma}{\delta\left(1-\sigma\right)\left(1-\kappa\right)-\left[1-\delta\left(1-\kappa\right)\right]\left(\sigma-\theta\right)\gamma}\right)$$

$$\overline{\pi}\left(\eta-1\right) \;=\; \left(1-\alpha\right)\mu\left(\frac{\delta\left(1-\sigma\right)\left(1-\kappa\right)-\left[1-\delta\left(1-\kappa\right)\right]\left(\sigma-\theta\right)\gamma}{\left(1-\delta\left(1-\kappa\right)\right)+\left(1-\kappa\right)\delta\sigma+\left(1-\delta\left(1-\kappa\right)\right)\sigma\gamma}\right)$$

$$\widehat{\eta} \;=\; 1+\left(\frac{\mu}{\overline{\pi}}\right)\left(\frac{\left(1-\alpha\right)\left[\delta\left(1-\sigma\right)\left(1-\kappa\right)-\left(1-\delta\left(1-\kappa\right)\right)\left(\sigma-\theta\right)\gamma\right]}{\left(1-\kappa\right)\delta\sigma+\left(1+\sigma\gamma\right)\left(1-\delta\left(1-\kappa\right)\right)}\right)$$

9   Appendix C: Numerical Methods[36]

There are 9 parameters that are embedded in the general model – n, α, ω, θ, κ, δ, q, γ, and λ.   The baseline simulation assumes: $(n,\alpha,\omega,\theta,\kappa,\delta,q,\gamma,\lambda)=(4,0,.75,0$

---

[36] The Mathematica code that generates the equilibrium cartel rates for the baseline case is available at: http://academic.csuohio.edu/changm/main/research/papers/CLPcodeA.pdf.

or 1, .05, .85, .2, .5, 1), where $\theta = 0$ with leniency program and $\theta = 1$ without leniency program.

For the probability function, $p(\lambda L + R)$, we consider two forms, linear and non-linear, as specified below:

$$p(\lambda L + R) = \begin{cases} \max\{c - m(\lambda L + R), 0.05\}, & \text{where } c < 1, \\ \frac{\tau}{\xi + \upsilon(\lambda L + R)^\rho}, & \text{where } \upsilon > 0, \, \rho \geq 1, \, \tau \in (0, 1], \, \xi \geq \tau \end{cases}$$

For the first specification, the probability decreases linearly with the caseload until it reaches its minimum value of 0.05. The second specification assumes a non-linear (concave then convex) relationship between the CA's caseload and the probability of success. Hence, the linear specification has two parameters, c and m, while the non-linear specification has four parameters, τ, ξ, υ, and ρ. The baseline simulation assumes: $(c, m) = (.8, 40)$ and $(\tau, \xi, \upsilon, \rho) = (1, 1, 1000, 1.4)$.

We assume a log-normal distribution, $LN(\mu, \sigma^2)$, for the two distributions, $H(\pi)$ and $G(\eta)$, where $(\mu, \sigma) = (0, 1.5)$ for $H(\pi)$ and $(\mu, \sigma) = (1, 1.5)$ for $G(\eta)$. The lower and upper bounds for the distributions are: $(\underline{\pi}, \overline{\pi}) = (1, \infty), =$ and $(\underline{\eta}, \overline{\eta}) = (1.1, \infty)$.

The numerical problem has a nested structure. Given a value of r, the underlying problem is to find a fixed point, $\sigma^*(r)$, to $\sigma = q \times r \times p(\lambda L(\sigma) + R(\sigma))$, where $L(\sigma)$ is the mass of cartel cases generated by the leniency program and $R(\sigma)$ is the mass of non-leniency cartel cases. Note that both L and R depend on σ, the endogenous probability of paying penalties, which affects the incentive compatibility of collusion.

The procedure for finding $\sigma^*(r)$ begins by specifying an initial value for σ. For each η, we need to solve for a fixed point to $\psi(Y, \sigma, \eta)$,

$$Y^*(\sigma, \eta) = \psi(Y^*(\sigma, \eta), \sigma, \eta).$$

As there may be multiple fixed points, we use the Pareto criterion to select among them and thus choose the largest fixed point. Since $\psi(Y, \sigma, \eta)$ is increasing and $\psi(\mu, \sigma, \eta) < \mu$ then, by setting $Y^0 = \mu$ and iterating on $Y^{t+1} = \psi(Y^t, \sigma, \eta)$, this process converges to the largest fixed point, $Y^*(\sigma, \eta)$.

In computing the stationary distribution of cartels, we need to take the step of computationally searching for $\widehat{\eta}(\sigma)$ which is the smallest industry type for which collusion is not incentive compatible for any market condition. $\widehat{\eta}(\sigma)$ is defined by: $Y^*(\sigma, \eta) > \alpha\mu$ for $\underline{\eta} < \eta \leq \widehat{\eta}(\sigma)$ and $Y^*(\sigma, \eta) = \alpha\mu$ for $\eta > \widehat{\eta}(\sigma)$. To perform this step, we set $\underline{\eta} = 1.1$ and $\overline{\eta} = 10$ and use a 1,000 element finite grid of values for η, denoted $\Gamma(\underline{\eta}, \overline{\eta})$. $\widehat{\eta}(\sigma)$ is located by applying the iterative bisection method on $\Gamma(\underline{\eta}, \overline{\eta})$. As part of the bisection method, $\overline{\eta}$ needs to be set at a sufficiently high value so that $Y^*(\sigma, \overline{\eta}) = \alpha\mu$. Once having identified $\widehat{\eta}(\sigma)$ and using $Y^*(\sigma, \eta)$, $\phi^*(\sigma, \eta)$ is calculated for a finite grid over $[\underline{\eta}, \widehat{\eta}(\sigma)]$. These values are then used in computing $L(\sigma)$ and $R(\sigma)$. The integration uses the Newton-Cotes quadrature method with the trapezoid rule (see Miranda and Fackler, 2002).

Choosing an initial value for σ and using our derived expressions for $L(\sigma)$ and $R(\sigma)$, we then compute:

$$\widehat{\sigma}(r) = q \times r \times p(\lambda L(\sigma) + R(\sigma))$$

given a specification (linear or non-linear) for $p(\lambda L(\sigma) + R(\sigma))$. After specifying a tolerance level ϵ, if $|\sigma - \widehat{\sigma}(r)| > \epsilon$ then a new value for σ is selected using the iterative bisection method. Note that once a new value for σ is specified, the entire preceding procedure must be repeated. This procedure is repeated until the process converges to the fixed point value of $\sigma^*(r)$ such that $|\sigma^*(r) - \widehat{\sigma}(r)| \leq \epsilon$. ϵ is set at .0002.

Given the equilibrium probability of paying penalties, $\sigma^*(r)$, we can calculate the equilibrium cartel rate, $C(\sigma^*(r))$, mass of leniency cases, $L(\sigma^*(r))$, and mass of non-leniency cases, $R(\sigma^*(r))$.

Note that the optimal competition policy, r^*, is the value of r that minimizes the equilibrium rate of cartels, $C(\sigma^*(r))$. To numerically derive r^*, we allow $r \in \{0, .1, ..., 1\}$ and perform the procedures described above for each of these values and identify the one that generates the minimum cartel rate.

The computational model, as described above, was run for both the linear and non-linear specifications of the success probability function, $p(\lambda L + R)$. In addition to the baseline parameter values, we considered a wide variety of parameter values off of the baseline in order to check for the robustness of the main properties identified in the paper. Specifically, for both the linear and non-linear $p(\lambda L + R)$ we considered $\gamma \in \{0.7, 0.8, 0.9\}$. Further robustness checks were performed for the non-linear $p(\lambda L + R)$ for the following parameter values off of the baseline: $\rho \in \{1.2, 1.4, 1.6\}$; $\gamma \in \{0.3, 0.7, 2.0\}$; $\lambda \in \{0.6, 0.8\}$; $\upsilon \in \{100, 500\}$; $n = 2$ (and, hence, $\omega = 0.5$ or $1)$[37]; $\alpha \in \{0.2, 0.5\}$; $\kappa = 0.1$; $\delta \in \{0.75, 0.95\}$. For all these parameter values, the numerical results are consistent with the properties stated in the paper: i) a leniency program can lower or raise the cartel rate; ii) the change in average cartel duration from a leniency program is decreasing in the industry type, η; and iii) $\widehat{\eta}$ is (generally) lower when there is a leniency program – $\widehat{\eta}_L < \widehat{\eta}_{NL}$.

---

[37]Note that $\omega = \frac{n-1+\theta}{n}$, where $\theta = 0$ with a leniency program and $\theta = 1$ without a leniency program. Hence, for $n = 2$, $\omega = 0.5$ with a leniency program and $\omega = 1$ without a leniency program.

References

[1] Apesteguia, Jose, Martin Dufwenberg, and Reinhard Selten, "Blowing the Whistle," *Economic Theory*, 31 (2007), 143-166.

[2] Aubert, Cecile, Patrick Rey, and William Kovacic, 2003, "The Impact of Leniency and Whistle-Blowing Programs on Cartels," *International Journal of Industrial Organization*, 24 (2006), 1241-1266.

[3] Bigoni, Maria, Sven-Olof Fridolfsson, Chloé Le Coq, and Giancarlo Spagnolo, "Fines, Leniency, and Rewards in Antitrust," *RAND Journal of Economics*, 43 (2012), 368-390.

[4] Borrell, Joan-Ramon, Juan Luis Jiménez, and Carmen García, "Evaluating Antitrust Leniency Programs," Universitat de Barcelona, January 2012.

[5] Brenner, Steffen, "An Empirical Study of the European Corporate Leniency Program," *International Journal of Industrial Organization,* 27 (2009), 639-645.

[6] Chen, Joe and Joseph E. Harrington, Jr., "The Impact of the Corporate Leniency Program on Cartel Formation and the Cartel Price Path," in *The Political Economy of Antitrust*, V. Ghosal and J. Stennek, eds., Elsevier, 2007.

[7] Chen, Zhijun and Patrick Rey, "On the Design of Leniency Programs," University of Toulouse, 2007.

[8] Choi, Jay Pil and Heiko Gerlach, "Global Cartels, Leniency Programs and International Antitrust Cooperation," Michigan State University, March 2010.

[9] Dijkstra, Peter T., Marco A. Haan, and Lambert Schoonbeek, "Leniency Programs and the Design of Antitrust: Experimental Evidence with Unrestricted Communication," University of Groningen, 2011.

[10] Ellis, Christopher J. and Wesley W. Wilson, "Cartels, Price-Fixing, and Corporate Leniency Policy: What Doesn't Kill Us Makes Us Stronger," University of Oregon, 2001.

[11] Friederiszick, Hans W. and Frank P. Maier-Rigaud, "Triggering Inspections Ex Officio: Moving Beyond a Passive EU Cartel Policy," *Journal of Competition Law and Economics*, 4 (2008), 89-113.

[12] Ghosal, Vivek and Joseph Gallo, "The Cyclical Behavior of the Department of Justice's Antitrust Enforcement Activity," *International Journal of Industrial Organization,* 19 (2001), 27-54.

[13] Harrington, Joseph E., Jr., "Behavioral Screening and the Detection of Cartels," in *European Competition Law Annual 2006: Enforcement of Prohibition of Cartels*, Claus-Dieter Ehlermann and Isabela Atanasiu, eds., Oxford: Hart Publishing, 2007.

[14] Harrington, Joseph E., Jr., "Optimal Corporate Leniency Programs," *Journal of Industrial Economics*, 56 (2008), 215-246.

[15] Harrington, Joseph E., Jr., "When is an Antitrust Authority not Aggressive Enough in Fighting Cartels?," *International Journal of Economic Theory*, 7 (2011a), 39-50.

[16] Harrington, Joseph E., Jr., "Corporate Leniency Programs when Firms have Private Information: The Push of Prosecution and the Pull of Pre-emption," Johns Hopkins University, 2011b (*Journal of Industrial Economics*, forthcoming).

[17] Harrington, Joseph E., Jr., "Corporate Leniency with Private Information: An Exploratory Example," in *Recent Advances in the Analysis of Competition Policy and Regulation*, J. Harrington and Y. Katsoulakos, eds., Cheltenham, UK: Edward Elgar, 2012.

[18] Harrington, Joseph E., Jr. and Myong-Hun Chang, "Modelling the Birth and Death of Cartels with an Application to Evaluating Antitrust Policy," *Journal of the European Economic Association*, 7 (2009), 1400-1435.

[19] Hinloopen, Jeroen and Adriaan R. Soetevent, "Laboratory Evidence on the Effectiveness of Corporate Leniency Programs," *RAND Journal of Economics*, 39 (2008), 607-616.

[20] Hyytinen, Ari, Frode Steen, and Otto Toivanen, "Cartels Uncovered," Centre for Policy Economic Research, Discussion Paper No. 7761, March 2010.

[21] Jeffrey, R. L., "The Continuity of a Function Defined by a Definite Integral," *The American Mathematical Monthly*, 32 (1925), 297-299.

[22] Klein, Gordon, "Cartel Destabilization and Leniency Programs - Empirical Evidence," ZEW Discussion Paper No. 10-107, 2010.

[23] Kwoka, John, "Commitment to Competition: An Assessment of Antitrust Agency Budgets since 1970," *Review of Industrial Organization,* 14 (1999), 295-302.

[24] Miller, Nathan, "Strategic Leniency and Cartel Enforcement," *American Economic Review*, 99 (2009), 750-768.

[25] Miranda, Mario J. and Paul L. Fackler, *Applied Computational Economics and Finance*, Cambridge, Mass.: The MIT Press, 2002.

[26] Motta, Massimo and Michele Polo, "Leniency Programs and Cartel Prosecution," *International Journal of Industrial Organization*, 21 (2003), 347-379.

[27] Riley, Alan, "Developing Criminal Cartel Law: Dealing with the Growing Pains," *Competition Law Review*, 4 (2007), 1-6.

[28] Rotemberg, Julio J. and Garth Saloner, "A Supergame-Theoretic Model of Price Wars During Booms," *American Economic Review*, 76 (1986), 390-407.

[29] Spagnolo, Giancarlo, "*Divide et Impera*: Optimal Deterrence Mechanisms Against Cartels and Organized Crime," University of Mannheim, 2003.

[30] Spagnolo, Giancarlo, "Leniency and Whistleblowers in Antitrust," in *Handbook of Antitrust Economics*, Paolo Buccirossi, ed., Cambridge, Mass.: The MIT Press, 2008.Veljanovski, Cento, "European Commission Cartel Prosecutions and Fines, 1998-2007," Case Associates, 2007.

[31] Zhou, Jun, "Evaluating Leniency and Modeling Cartel Durations: Time-Varying Policy Impacts and Sample Selection," Bonn University, September 2011.

# Figure 1: Cartel Behavior with Linear $p(\lambda L + R)$

Percentage Change in Cartel Rates Due to LP

- $\gamma = .1$
- $\gamma = .2$
- $\gamma = .3$
- $\gamma = .4$
- $\gamma = .5$
- $\gamma = .6$
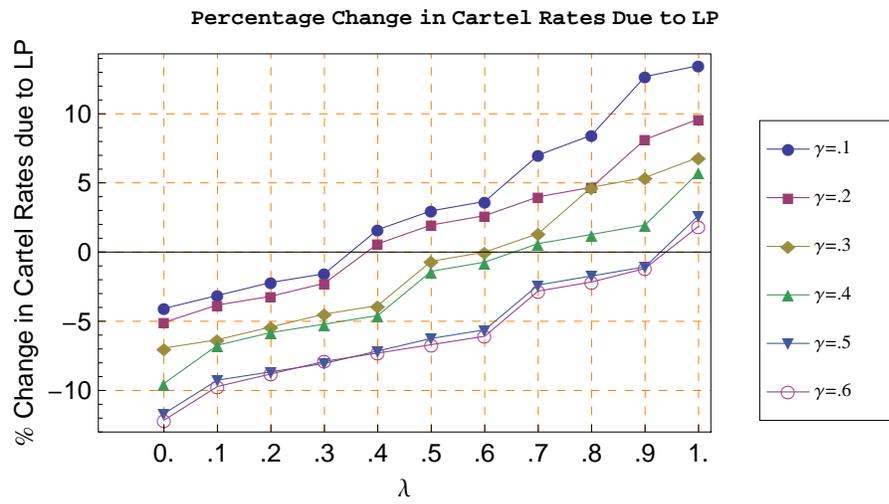
Figure 2: Cartel Behavior with Non-linear $p(\lambda L + R)$

Percentage Change in Cartel Rates Due to LP

- $\gamma = .1$
- $\gamma = .2$
- $\gamma = .3$
- $\gamma = .4$
- $\gamma = .5$
- $\gamma = .6$
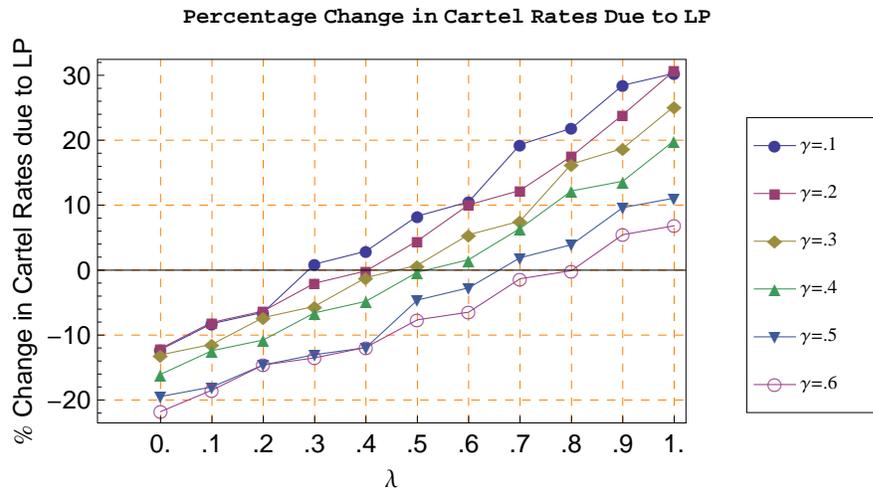
Figure 3: Change in average cartel duration conditional on η for linear $p(\lambda L + R)$ with $\lambda = 1.0$ and $\gamma \in \{.7, .8, .9\}$

(a) $\gamma = .7$ **and** $\lambda = 1.0$

$\hat{\eta}_L = 1.5272, \hat{\eta}_{NL} = 1.5539$
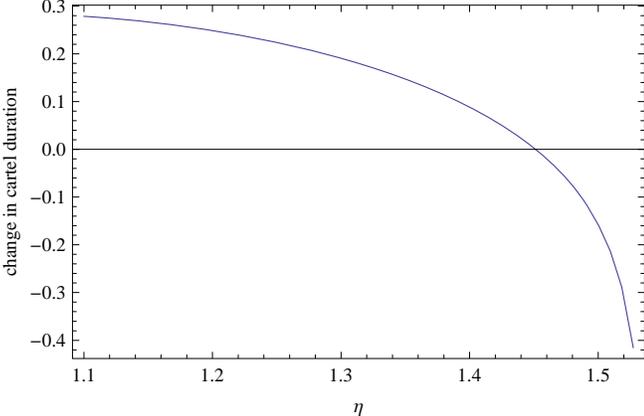
(b) $\gamma = .8$ **and** $\lambda = 1.0$

$\hat{\eta}_L = 1.5183, \hat{\eta}_{NL} = 1.5539$
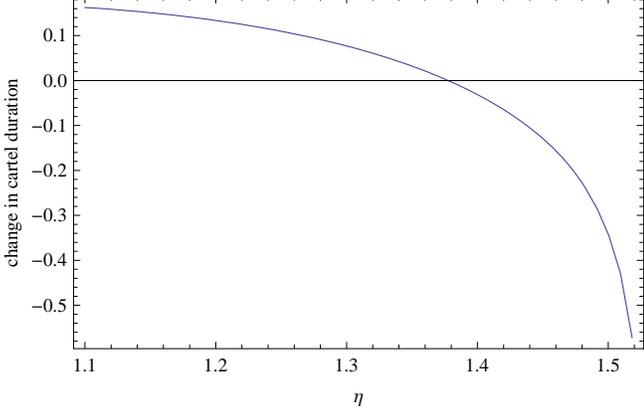
(c) $\gamma = .9$ **and** $\lambda = 1.0$

$\hat{\eta}_L = 1.5094, \hat{\eta}_{NL} = 1.5450$
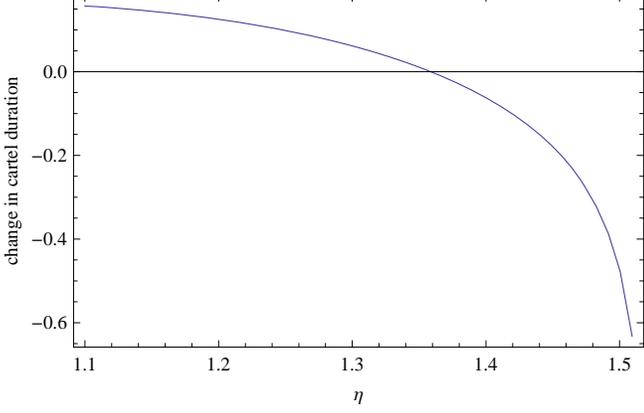
Figure 4: Change in average cartel duration conditional on η for non-linear $p(\lambda L + R)$ with $\lambda = 1.0$ and $\gamma \in \{.7, .8, .9\}$

(a) $\gamma = .7$ and $\lambda = 1.0$

$\hat{\eta}_L = 1.5183, \hat{\eta}_{NL} = 1.5361$
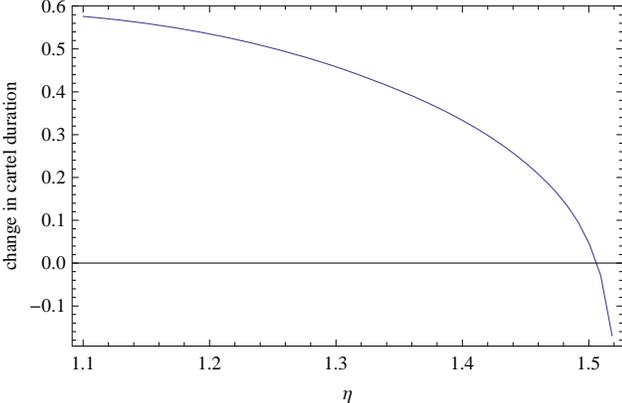
(b) $\gamma = .8$ and $\lambda = 1.0$

$\hat{\eta}_L = 1.5094, \hat{\eta}_{NL} = 1.5272$
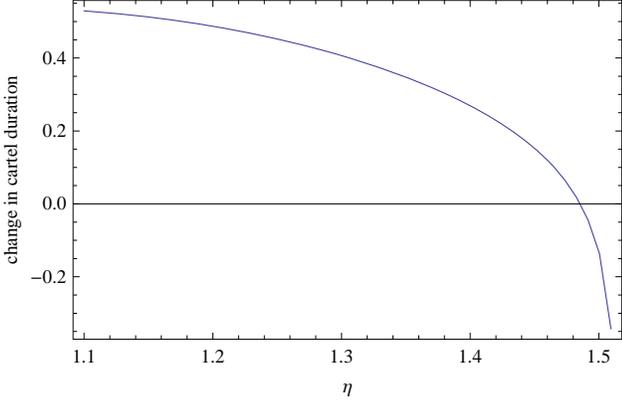
(c) $\gamma = .9$ and $\lambda = 1.0$

$\hat{\eta}_L = 1.4916, \hat{\eta}_{NL} = 1.5272$
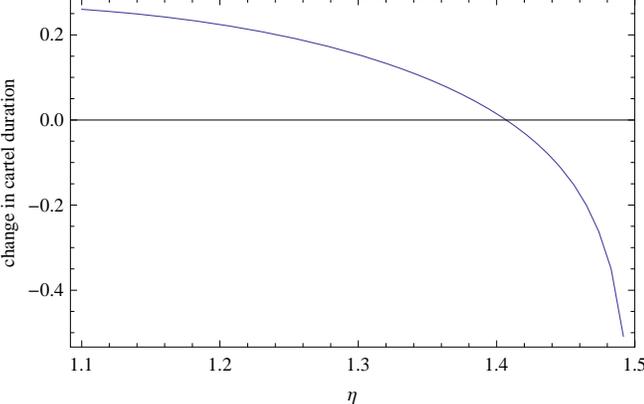
Figure 5a
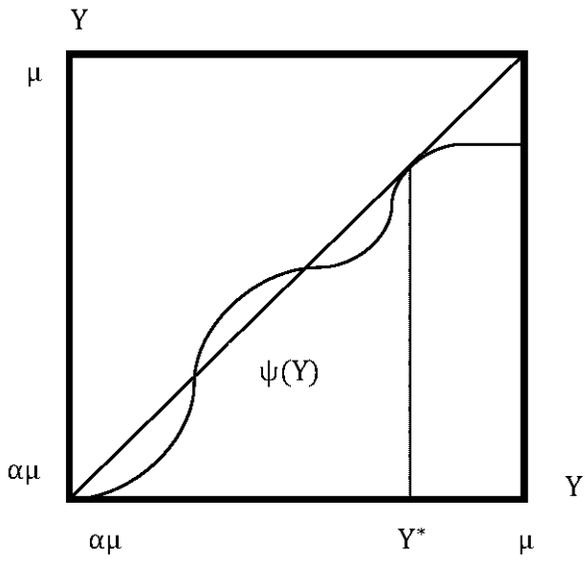
Figure 5b
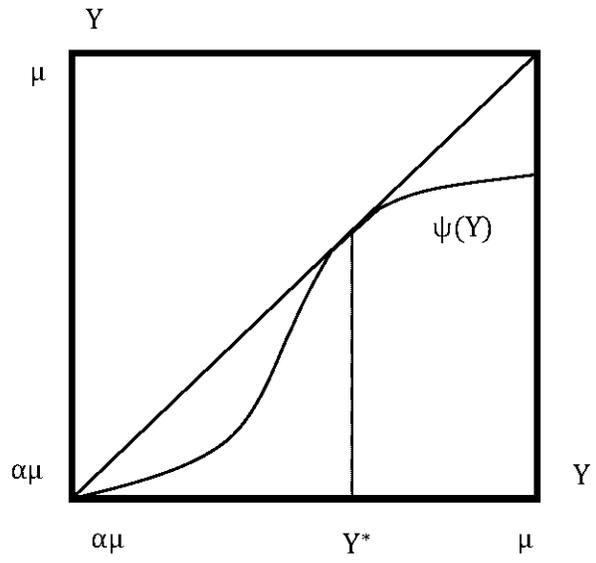
Figure 5c

Figure 6

Figure 7a

$\mu < A$

Y

μ

αμ

$\psi(Y)$

Y

$\alpha\mu = Y^*$ μ

Figure 7b

$A < \mu < B$

Y

μ

αμ   Y

$\psi(Y)$

$\alpha\mu = Y^*$ A   μ

Figure 7c

$B < \mu$

Y

μ

$\psi(Y)$

αμ   Y

$\alpha\mu = Y^*$ A   B   μ

Figure 7d

$B < \mu$

Y

μ

$\psi(Y)$

αμ   Y

αμ   A   B Y^* μ