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“Time Lotteries”

by

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Time Lotteries*

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Abstract

We study preferences over lotteries that pay a specific prize at uncertain dates. Expected Utility with convex discounting implies that individuals prefer receiving $\$x$ in a random date with mean t over receiving $\$x$ in t days for sure. Our experiment rejects this prediction. It suggests a link between preferences for payments at certain dates and standard risk aversion. Epstein-Zin (1989) preferences accommodate such behavior, and fit the data better than a model with probability weighting. We thus provide another justification for disentangling attitudes toward risk and time, as in Epstein-Zin, and suggest new theoretical restrictions on its key parameters.

Key words: Discounted Expected Utility, Epstein-Zin preferences, Non-Expected Utility, Risk aversion towards time lotteries

JEL: C91, D81, D90

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1 Introduction

Suppose you are offered a choice between (i) receiving $\$x$ in period t for sure, or (ii) receiving $\$x$ in a random period \tilde{t} with mean t . For example, you can receive \$100 in 10 weeks for sure, or \$100 in either 5 or 15 weeks with equal probability. Both lotteries pay the same amount and have the same expected delivery date. However, the delivery date is known in the first lottery and is uncertain in the second one. Suppose also that if you choose option (ii), the exact payment date will be revealed immediately, eliminating any planning issues. Which would you choose?

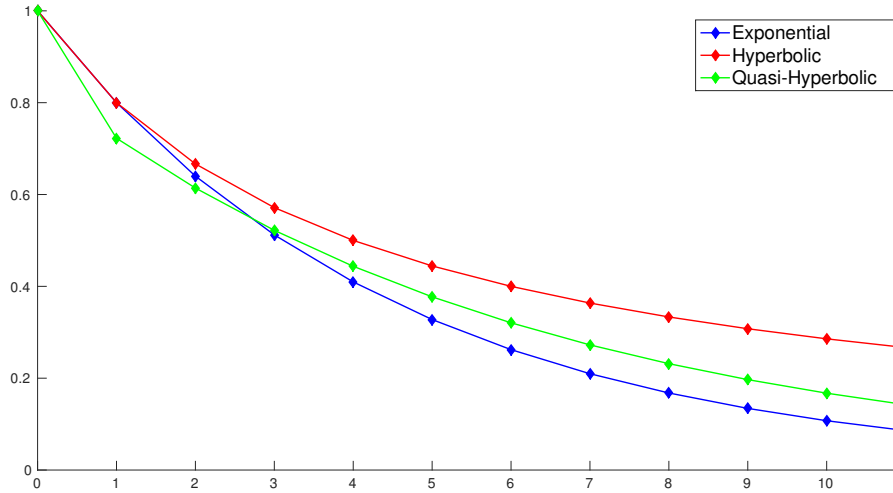
In this paper we study preferences over time lotteries: lotteries that pay the same prize at uncertain future dates, with the uncertainty fully resolved immediately after the choice is made.¹ Uncertainty about timing has a bearing on many real life choices. For example, when one decides whether to invest in a project that is certain to start paying dividends in 5 years rather than in one which starts payments within an *average* of 5 years. Another example is whether to pay more to guarantee that the delivery date of an online purchase (like a book from Amazon) is t days from now, rather than t days on average.

The starting point of our analysis is the observation that the ubiquitous model of time preferences – Discounted Expected Utility with convex discounting – imposes a specific direction of these preferences: independently of the values of x and t , when offered a choice between options (i) and (ii) above, subjects should *always* pick the option with an uncertain payment date (ii); that is, they must be *risk seeking over time lotteries* (RSTL). To see this, recall that the standard way to evaluate a consumption path that pays c in each period and $c + x$ in some period t is given by $\sum_{\tau \neq t} D(\tau) u(c) + D(t) u(c + x)$, where u is a time-independent utility function over outcomes and D is a decreasing discount function. If the consumption path is random, overall utility is obtained by taking expectations, leading to the Discounted Expected Utility (DEU) model. As long as the discount function D is convex, Jensen’s inequality implies that option (ii) is preferred. Note that this is independent of the curvature of the utility function u , as the payment is the same in both options. Since virtually *all* discount functions used in economics are convex (including exponential, hyperbolic, and quasi-hyperbolic; see Figure 1) and since non-convex discounting is possible at only finitely many points (as we discuss in Section 2), this is a fundamental feature of the standard model.

We contend that this prediction of DEU is too strong: in an incentivized experiment, we find that most subjects are not RSTL. Instead, subjects normally pick lotteries with known payment dates, exhibiting risk aversion over time lotteries (RATL). Our theoretical contribution is to show that two well-known generalizations of the standard model — the separation of risk aversion and time preference in Epstein and Zin (1989) (henceforth EZ) and a discounted non-Expected Utility model with probability weighting — can both account for this behavior. We then use

¹The notion of time lotteries is introduced in Chesson and Viscusi (2003) (termed “lottery-timing risk aversion”), and also analyzed in Onay and Öncüler (2007); we discuss the relation with these papers below.

Figure 1: The exponential, hyperbolic, and quasi-hyperbolic discount functions



Notes. The exponential, hyperbolic, and quasi-hyperbolic discount functions are, respectively, $D(t) = \delta^t$, $D(t) = \frac{1}{1+\gamma t}$, and $D(t) = \beta\delta^t$ for $t \geq 1$ (with $D(0) = 1$), where $\beta, \delta \in (0, 1)$ and $\gamma > 0$. They are all convex.

our experimental findings to demonstrate that EZ better fits the data. Our results provide a new justification for the separation of attitudes toward risk and intertemporal substitution in EZ and also suggest new theoretical restrictions on its parameters.

According to EZ, time lotteries are evaluated by first computing the discounted utility of each consumption path in its support. Then, unlike in DEU, these discounted utilities are aggregated *non-linearly*; the certainty equivalent of the lottery over them is computed in a way that depends on the curvature of a function v , which captures the individual's risk aversion and is different from the function u used in evaluating deterministic consumption paths. If the individual is risk averse enough (v is sufficiently concave), then she will prefer a payment in a known date to a payment with an uncertain but mean preserving date. Thus, EZ predicts a correlation between risk aversion over time lotteries and standard atemporal risk aversion, since both are affected by the same parameter.

Another way to accommodate some risk aversion over time lotteries, also suggested in Onay and Öncüler (2007), is to relax Expected Utility by aggregating the discounted utilities using expectation with respect to distorted probability weights. This Discounted Probability Weighting Utility model (DPWU) predicts a preference for known payment dates only among subjects who violate Expected Utility by sufficiently underweighting the probabilities of good outcomes.

We conducted an experiment to test if subjects are risk seeking over time lotteries, and, if not, to separate between the two explanations above. We first asked subjects to choose between pairs of time lotteries, where the distribution of dates in one option was a mean preserving spread of that in the other. We then measured standard time preferences. Lastly, we asked questions

on regular risk preferences to measure subject’s atemporal risk aversion, as well as violations of Expected Utility and probability weighting. The following are our key findings:

1. Only a small number of subjects (less than 7%) are RSTL and the majority can be classified as RATL. When both options have random payment dates, most subjects still prefer the less risky ones (in the sense of mean preserving spreads), which suggests that they are not simply attracted to certainty.
2. A large majority of subjects (82%) exhibit convex discounting. The above result (1) remains unchanged if we restrict the analysis to these subjects.
3. Consistently with EZ, preferences for known payment dates are strongly related to atemporal risk aversion.
4. In contrast to DPWU, preferences for known payment dates are unrelated to violations of Expected Utility. Subjects who do not underweight probabilities still prefer known payment dates. The same is true if we focus only on those who abide by Expected Utility theory in atemporal risky choices (i.e., we eliminate those who both underweight and overweight probabilities). In addition, a regression analysis shows that the preference for known payment dates is unrelated to the degree of probability weighting.

Overall, our experimental results show the prevalence of RATL (contradicting DEU), and support the generalization based on EZ, but not the one based on probability weighting. This is of particular relevance because EZ is a widely-used model, especially in macroeconomics and finance. The common justification for adopting EZ is that it allows for two different parameters to govern attitudes toward risk and intertemporal substitution – an additional degree of freedom that has proved particularly effective in matching empirical data.² Behaviorally, it is sometimes justified as it allows for a preference for early rather than late resolution of uncertainty. Based on our results presented above, we suggest an additional reason to adopt this model: it permits a wider range of preferences over time lotteries, allowing decision makers to prefer payments with a known date.

The impossibility of DEU to accommodate different attitudes towards time lotteries can be understood with an analogy to the classic work of Yaari (1987). Within the (atemporal) Expected Utility framework, diminishing marginal utility of income and risk aversion are bonded together. But, as Yaari argues, these two properties are “horses of different colors” and hence, as a fundamental principle, a theory that keeps them separate is desirable. We show an analogous property in a temporal setting. Convex discounting, which is a property of *pure time preferences*, necessarily implies RSTL. There is no fundamental reason why the two notions should be related

²See, for example, Bansal and Yaron (2004) and Chen et al. (2013).

and, in fact, we find their equivalence even more troubling than the equivalence pointed out by Yaari. This is because while diminishing marginal utility of income and risk aversion relate to two different phenomena, they are both reasonable properties of preferences. In our case, while decreasing willingness to wait (convex discounting) is a plausible behavioral property, supported by our experimental data, it seems that most people are not RSTL.

This paper is related to the literature on the interaction between time and risk.³ Chesson and Viscusi (2003) introduce the idea of time lotteries, analyze the case of standard DEU with exponential discounting, and argue that uncertainty aversion over outcome timing should be correlated with uncertainty aversion over probabilities of outcomes. They conducted a hypothetical survey on business owners, and found that 30 percent of the subjects dislike uncertainty in the timing of an outcome. Onay and Öncüler (2007) generalize their theoretical result, pointing out that (what we call) RSTL holds in DEU for any convex discounting (their analysis also focuses on the roles of gains and losses). They conducted an un-incentivized survey, with large hypothetical payments, and also found that subjects dislike uncertainty in timing. They link this to violations of Expected Utility due to probability distortions. Unlike these two papers, we show that EZ can also theoretically account for a preference for sure dates. Moreover, we conducted an actual (incentivized) experiment in which we confirmed the prevalence of preferences for known payment dates. When we compared possible explanations, we found support for the one based on EZ. Eliaz and Ortoleva (forthcoming) studied the case in which the payment date is ambiguous, as opposed to risky. They found that the majority of subjects remain averse to this ambiguity, but the proportion is much smaller than that of aversion to ambiguous payments.

Other papers in the literature on the relation between time and risk mostly focus on different issues and do not analyze attitude towards time lotteries. For example, Halevy (2008) suggests a link between present bias (Strotz, 1955) and the certainty effect (Kahneman and Tversky, 1979) based on the idea that the present is known while any future plan is inherently risky. He shows that the two ideas can be bridged, if the individual evaluates stochastic streams of payments using specific non-Expected Utility functionals. Andreoni and Sprenger (2012) challenge a property of DEU, according to which intertemporal allocations should depend only on relative intertemporal risk. They conducted an experiment that focused on common-ratio type questions, applied to intertemporal risk. Their result indicates that subjects display a preference for certainty when it is available, but behave closely in line with the predictions DEU when only uncertain (future) prospects are considered. Andersen et al. (2008), Epper et al. (2011), and Dean and Ortoleva (2015) experimentally studied the relation between time and risk preferences. They found that time preferences are correlated with the curvature of the utility function, as predicted by the standard model; their results on the relation with violations of Expected Utility are mixed.

The remainder of the paper is organized as follows: Section 2 formally shows that DEU

³See Epper and Fehr-Duda (2015) and references therein.

implies risk seeking towards time lotteries. Section 3 introduces the two possible generalizations that can allow for a wider range of behavior — EZ and discounted probability weighting — and discuss their empirical predictions. Section 4 presents the experimental design and its results. The appendix includes proofs of the formal results and the experimental details.

2 DEU and Attitudes towards Time Lotteries

We assume time to be discrete and identify the set of possible times with the set of natural numbers. A consumption path is a sequence $\mathbf{c} = (c_1, c_2, \dots, c_t, \dots)$ that specifies consumption in each period $c_t \in \mathbb{R}_+$.

Our analysis focuses on a particular type of lotteries over consumption paths, which we call *time lotteries*. Fix a base level of per-period consumption c . For each monetary prize $x \in \mathbb{R}_+$ and time $t \in \mathbb{N}$, let (x, t) denote the consumption path that pays $c + x$ in period t and c in any other period. A time lottery $p_x = \langle p_x(t), (x, t) \rangle_{t \in \mathbb{N}}$ is a finite probability measure over $\{x\} \times \mathbb{N}$, that is, a lottery that yields an additional payment of x in period t with probability $p_x(t)$.⁴ The degenerate lottery that yields (x, t) for sure is denoted by $\delta_{(x, t)}$. Let \mathcal{P}_x denote the set of all time lotteries with payment x . Preferences are defined over $\mathcal{P} := \bigcup_{x \in X} \mathcal{P}_x$ and are represented by some function $V : \mathcal{P} \rightarrow \mathbb{R}$.

We say that preferences are *risk averse towards time lotteries (RATL)* if, for every $x \in \mathbb{R}_+$ and every $p_x \in \mathcal{P}_x$ with $\sum_{\tau} p_x(\tau) \tau = t$,

$$V(\delta_{(x, t)}) \geq V(p_x). \quad (1)$$

They are *risk seeking towards time lotteries (RSTL)* if the sign in (1) is reversed.

According to the Discounted Expected Utility model (DEU), the value of each time lottery p_x is given by

$$V_{DEU}(p_x) = \sum_t p_x(t) \left[\sum_{\tau \neq t} D(\tau) u(c) + D(t) u(c + x) \right],$$

where $u : X \rightarrow \mathbb{R}_+$ is a continuous and strictly increasing utility function, and $D : \mathbb{N} \rightarrow (0, 1]$ is a strictly decreasing discount function.

We say that a discount function is *discretely convex* if D is a convex function when defined,

⁴To avoid confusion, let us emphasize that time lotteries are fundamentally different from lotteries that pay an uncertain amounts in different periods. For example, the time lottery that pays \$1 in either periods 1 and 3 with equal probabilities is very different from the lotteries that pay \$0 or \$1 with 50 percent chance in periods 1 and 3. With the former, the decision maker always gets exactly \$1, although she does not know when. With the latter, she may get a total of \$0, \$1, or \$2.

that is, if for all $t_1, t_2 \in \mathbb{N}$ and $\alpha \in (0, 1)$,

$$\alpha D(t_1) + (1 - \alpha) D(t_2) \geq D(\alpha t_1 + (1 - \alpha) t_2)$$

whenever $\alpha t_1 + (1 - \alpha) t_2 \in \mathbb{N}$.

The following proposition establishes the relationship between attitudes towards time lotteries and the convexity of the discount function. (The first part of our result, confined to two prizes coded as “gains,” is stated as Hypothesis 1 in Onay and Öncüler 2007.)

Proposition 1. *Under DEU, preferences are RSTL if and only if D is discretely convex. Moreover, they cannot be RATL.*

Proof. First, we show that preferences are RSTL (RATL) if and only if D is discretely convex (concave). (A discount function is discretely concave if $-D$ is discretely convex.) The value of $\delta_{(x, \bar{t})}$ is

$$V_{DEU}(\delta_{(x, \bar{t})}) = \sum_{\tau \neq \bar{t}} D(\tau) u(c) + D(\bar{t}) u(c + x),$$

whereas the value of the time lottery $p = \langle p_x(t), t \rangle_{t \in \mathbb{N}}$ with $\sum_t p_x(t) t = \bar{t}$ is

$$V_{DEU}(p) = \sum_t p_x(t) \left[\sum_{\tau \neq t} D(\tau) u(c) + D(t) u(c + x) \right]$$

With algebraic manipulations give:

$$V_{DEU}(p) \geq V_{DEU}(\delta_{(x, \bar{t})}) \Leftrightarrow \left[\sum p_x(t) D(t) - D(\bar{t}) \right] [u(c + x) - u(c)] \geq 0,$$

which, because u is strictly increasing, holds if and only if D is convex.

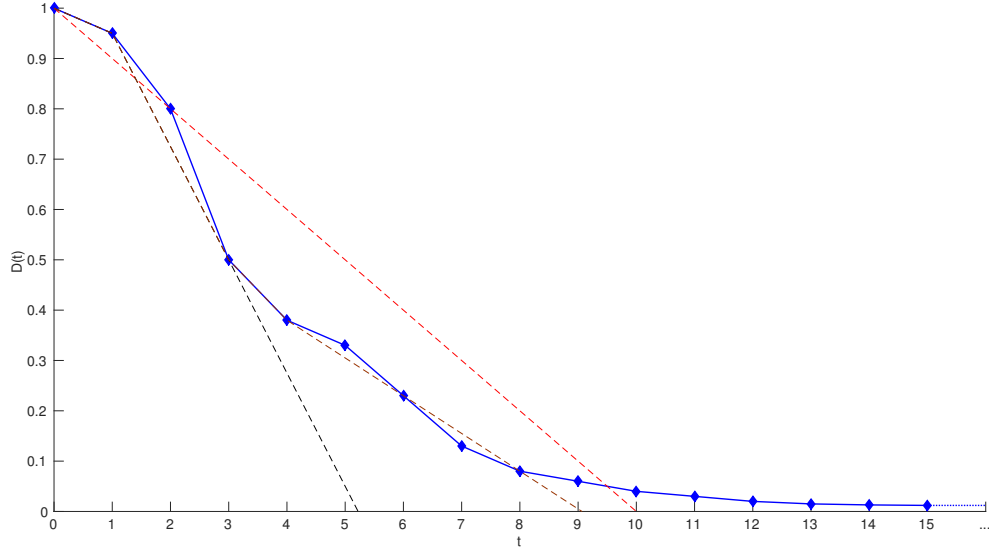
Next, we show that D cannot be discretely concave. Suppose D is discretely concave, so that

$$D(t) \leq D(1) + (t - 1) [D(2) - D(1)].$$

Taking $t \geq \frac{2D(1) - D(2)}{D(1) - D(2)}$ and using the fact that D is strictly decreasing, we obtain $D(t) < 0$, which contradicts the fact that the discount function is positive. \square

The first part of Proposition 1 states that if the discount function is convex, a DEU decision maker *must* be RSTL. Importantly, this result does not rely on the curvature of the utility function u since all options involve the same payments, although in different periods. But is it plausible to assume that the discount function is convex? As we pointed out in the introduction, virtually all discount functions used in economics are convex (see Figure 1). Indeed, convexity of the discount function has a natural behavioral interpretation: it means that time delays are less costly the further away in time they occur. Therefore, with standard discount functions, DEU leaves no degrees of freedom on the risk attitude towards time lotteries.

Figure 2: Discount functions are locally convex in all but a finite number of periods



Notes. Dotted lines are the secants going through the points adjacent to the dates where D is locally concave. To be globally concave, D would have to be below these secants, crossing the horizontal axis.

The second part of Proposition 1 states that discount functions cannot be (globally) concave, implying that we cannot have RATL with DEU. Figure 2 illustrates the intuition behind this result. The discount function must be decreasing. Concavity requires it to decrease at an increasing rate, meaning that the discount function at all periods greater than $t + 1$ must lie below the secant passing through $D(t - 1)$ and $D(t + 1)$. If the discount function were concave, it would eventually cross the horizontal axis, contradicting the fact that discount functions are positive.

In light of Proposition 1, we ask whether DEU can satisfy weaker, local versions of RATL. Since time lotteries have two dimensions (time and prizes), we consider two different notions of local RATL. We say that preferences are *locally risk averse towards time lotteries at prize x* if

$$V(\delta_{(x,t)}) \geq V(p)$$

for every $p \in \mathcal{P}_x$ with $\sum_{\tau} p_x(\tau) \tau = t$. They are *locally risk averse towards time lotteries at time t* if the sure payment at t is preferred to a random payment occurring at either $t - 1$ or at $t + 1$ with equal probabilities, that is,

$$V(\delta_{(x,t)}) \geq V(\langle 0.5, (x, t - 1); 0.5, (x, t + 1) \rangle)$$

for every $x \in \mathbb{R}_+$. As before, we say that preferences are locally risk seeking at either x or t if

the reverse inequalities hold.⁵

Proposition 2 below shows that even these weaker versions of RATL are inconsistent with DEU. Thus, even if we were willing to abandon convexity, it would be of limited help. Since the previous discussion did not rely on varying the prize x , it follows that DEU also cannot accommodate local RATL at any x . Moreover, as Figure 2 illustrates, local concavity can hold only at a small number of periods. Therefore, preferences are generically locally RSTL in time, in the sense that they are locally RATL in only finite number of dates.

Proposition 2. *Under DEU, there is no x at which preferences are locally RATL. Moreover, the set of periods in which preferences are locally RATL at t is finite.*

Proof. See Appendix A. □

In the following section, we will see two theories, each of them allowing for one of these notions of local RATL to hold.⁶

3 Beyond DEU: Epstein Zin Preferences and Probability Weighting

We now present two generalizations of DEU that allow more flexible attitudes towards time lotteries. The first one disentangles attitudes towards risk from intertemporal substitution, leading to the model of Epstein and Zin (1989). The second replaces objective probabilities by decision weights. We show that both models can capture preferences for known payment dates and discuss their implications. In the next section, we will evaluate each of them using data from our experiment.

3.1 Separating Time and Risk Preferences: the model of Epstein Zin

Epstein and Zin (1989) (EZ) introduced a general class of preferences over stochastic consumption paths, defined recursively over this period's known consumption and the certainty equivalent of next period's utility. In the most popular version of EZ, lotteries over consumption paths are evaluated using the recursive formula:

⁵While neither notion of local RATL is contained in the other, being locally RSTL in any of them prevents the decision maker from being locally RATL in the other.

⁶The analysis thus far does not permit individuals to smooth consumption by redistributing their prize over time. In Appendix B, we allow the decision maker to costlessly borrow and save. We show that $\delta_{(x,t)}$ is preferred over $\langle 0.5, (x, t-1); 0.5, (x, t+1) \rangle$ if and only if the decision maker is sufficiently risk averse. However, quantitatively, choosing the safe lottery requires absurdly high risk aversion or extremely large prizes. For example, suppose the (market) rate of discount is 0.9. Then, an individual with a million dollars in discounted lifetime earnings and a constant coefficient of relative risk aversion of 10 would prefer the safe lottery only if the prize exceeds \$123,500.

$$V_t = \left\{ (1 - \beta) c_t^{1-\rho} + \beta [E_t (V_{t+1}^{1-\alpha})]^{\frac{1-\rho}{1-\alpha}} \right\}^{\frac{1}{1-\rho}}, \quad (2)$$

where c_t denotes consumption at time t , $\alpha > 0$ is the coefficient of relative risk aversion, and $\rho > 0$ is the inverse of the elasticity of intertemporal substitution. EZ boils down to DEU whenever $\alpha = \rho$. A well-known advantage of this model is that it separates the roles of risk aversion and the elasticity of intertemporal substitution – which must be the inverse of one another with DEU. This additional degree of freedom has proved to be particularly useful in applied work, and this model is widely used in macroeconomics, asset pricing, and portfolio choice. From a behavioral perspective, this generalization of DEU also allows subjects to express preferences for early or late resolution of uncertainty. One message from our analysis below is that separating risk aversion and the elasticity of intertemporal substitution also allows accommodating some risk aversion over time lotteries. This feature, in turns, suggests new theoretical restrictions on the values of the parameters α and ρ .

Given the simple structure of time lotteries, in which all uncertainty about future consumption is resolved immediately, the value of a time lottery $p \in \mathcal{P}_x$ using equation (2) is

$$V_{EZ}(p) = \left\{ (1 - \beta) c^{1-\rho} + \beta [E_p (V^{1-\alpha})]^{\frac{1-\rho}{1-\alpha}} \right\}^{\frac{1}{1-\rho}}, \quad (3)$$

where $E_p(\cdot)$ denotes the expectation with respect to the measure p .⁷ If we let $\lambda := \frac{c+x}{c} > 1$ denote the proportional increase in consumption from the prize, the continuation utility V is determined by

$$V(\delta_{(x,t)}) = [(1 - \beta) c]^{\frac{1}{1-\rho}} \left[\beta^t \lambda^{1-\rho} + \sum_{\tau \neq t} \beta^\tau \right]. \quad (4)$$

Consider a choice between the safe lottery $\delta_{(x,t)}$ and the risky lottery $\langle 0.5, (x, t - 1); 0.5, (x, t + 1) \rangle$. The next proposition establishes that EZ can accommodate a preference for the safe lottery over the risky one and determines the main comparative statics of these preferences given the parameters of the model:

Proposition 3. *Under EZ, for any β , ρ , and x , there exists $\bar{\alpha}_{\rho,\beta,x} > \max\{\rho, 1\}$ such that $V_{EZ}(\delta_{(x,t)}) > V_{EZ}(\langle 0.5, (x, t - 1); 0.5, (x, t + 1) \rangle)$ if and only if $\alpha > \bar{\alpha}_{\rho,\beta,x}$. Moreover, $\lim_{x \searrow 0} \bar{\alpha}_{\rho,\beta,x} = +\infty$.*

Proof. See Appendix A. □

Proposition 3 shows that, controlling for discounting β , elasticity of intertemporal substitution $1/\rho$, and the size of the prize x , more risk averse individuals are more likely to prefer the safe

⁷Note that in order to use equation (2), we think of any time lottery p as determining payoffs from next period on, preceded by a (known) consumption of c today.

lottery.⁸ That is, under EZ there is a connection between risk aversion over time lotteries and risk aversion over regular, atemporal lotteries. Moreover, the *risky* lottery is always preferred if the utility function is less concave than a logarithmic function ($\alpha < 1$), if $\alpha \leq \rho$, or if the prize is small enough.

To illustrate the nature of this trade-off, and why EZ allows for a preference for the safe lottery while DEU does not, consider the case of an infinite elasticity of intertemporal substitution ($\rho = 0$) and suppose the mean payment period is $t = 3$. Applying equation (3), the value of the safe lottery is simply the (per-period) discounted present value of consumption:

$$(1 - \beta) c \left[1 + \beta + \lambda \beta^2 + \frac{\beta^3}{1 - \beta} \right], \quad (5)$$

The value when choosing the risky lottery (the 50 : 50 mixture between $t = 2$ and $t = 4$) equals

$$(1 - \beta) c \left[1 + \beta \frac{\left(\lambda + \frac{\beta}{1 - \beta} \right)^{1 - \alpha} + \left(1 + \beta + \lambda \beta^2 + \frac{\beta^3}{1 - \beta} \right)^{1 - \alpha}}{2} \right]^{\frac{1}{1 - \alpha}}, \quad (6)$$

which is the (per-period) certainty equivalent of the atemporal lottery that pays the discounted present value of future payments, that is, it pays either $c \left(\lambda + \frac{\beta}{1 - \beta} \right)$, or $c \left(1 + \beta + \lambda \beta^2 + \frac{\beta^3}{1 - \beta} \right)$ with equal probabilities.

From (6), it follows that the value of the risky lottery is *decreasing* in the risk aversion α , while that of the safe lottery is unaffected. With risk neutrality ($\alpha = 0$), the risky lottery is preferred since it offers a higher expected discounted payment. As risk aversion α goes to infinity, however, the value of choosing the risky lottery decreases to that of receiving the worst possible outcome for sure – namely, the present discounted value from getting the prize at $t = 4$. Since receiving the prize at $t = 3$ is better than receiving it at $t = 4$, the safe lottery is preferred with extreme risk aversion. By monotonicity and continuity, there exists a unique cutoff $\bar{\alpha}_{0,\beta,x}$ separating the regions where safe lottery and the risky lottery are preferred. Since EZ coincides with DEU when $\alpha = \rho$, and DEU displays RSTL, the cutoff level of risk aversion $\bar{\alpha}_{\rho,\beta,x}$ must be greater than ρ .⁹ Further notice that the cutoff $\bar{\alpha}_{\rho,\beta,x}$ only depends on the ratio x/c and does not depend on the time distance t . This follows from the homogeneity of this version of EZ and the dynamic consistency property of these preferences.

A broader intuition of why EZ preferences need not be RSTL is the following. Under DEU, the decision maker evaluates the risky lottery by first computing the discounted utility of *each*

⁸That is, $\alpha' > \alpha$ implies that if the decision maker with coefficient of risk aversion α prefers the safe lottery over the risky one, so does the decision maker with α' (holding other parameters fixed).

⁹Starting with Kreps and Porteus (1978), a large literature has studied preferences over the timing of resolution of uncertainty. With EZ, early resolution of uncertainty is preferred if and only if $\alpha > \rho$ (Epstein et al., 2014). Proposition 3 then implies that this condition is also needed for the safe time lottery to be preferred.

possible consumption path, and then aggregating these values linearly, by taking expectation; no curvature, or distortion, is applied to this aggregation – in DEU all the curvature is applied when computing the value of each path. Instead, under EZ the decision maker also computes the discounted utility of each path, but then aggregate them *non-linearly*: she calculates the certainty equivalent of the lottery over them in a way that depends on the individual's risk aversion (as captured by the parameter α). If risk aversion is sufficiently high, then she will prefer the safe lottery.

At the same time, Proposition 3 also shows that $\lim_{x \searrow 0} \bar{\alpha}_{\rho, \beta, x} = +\infty$, which means that for any α, ρ and β , EZ preferences are locally RSTL at x whenever x is small enough. Therefore, with this formulation of EZ, preferences also cannot be locally RATL at any t (see footnote 5). Intuitively, this follows from the fact that the certainty equivalent of future paths is calculated according to Expected Utility (with constant relative risk aversion). Recall that Expected Utility implies that preferences are approximately risk neutral when stakes are very small (Segal and Spivak, 1990; Rabin, 2000). Then, since risk neutral preferences are RSTL, the risky lottery is preferred if the prize x is small enough.

We conclude with a brief discussion on the parameter restrictions on EZ implied by Proposition 3. Figures 4a and 4b present the loci of points separating the regions where the safe and risky lotteries are preferred for a discount parameter of $\beta = 0.9$. Points above each curve correspond to parameters that favor the safe lottery; below each curve, the risky lottery is preferred. Notice that the region where the safe lottery is preferred increases with the prize x and with the elasticity of intertemporal substitution (i.e., it decreases with ρ). For example, with $x = c$, $\rho = 0.6$, and $\beta = 0.9$, the safe lottery is chosen as long as α is at least 15.

3.2 Discounted Probability Weighting Utility

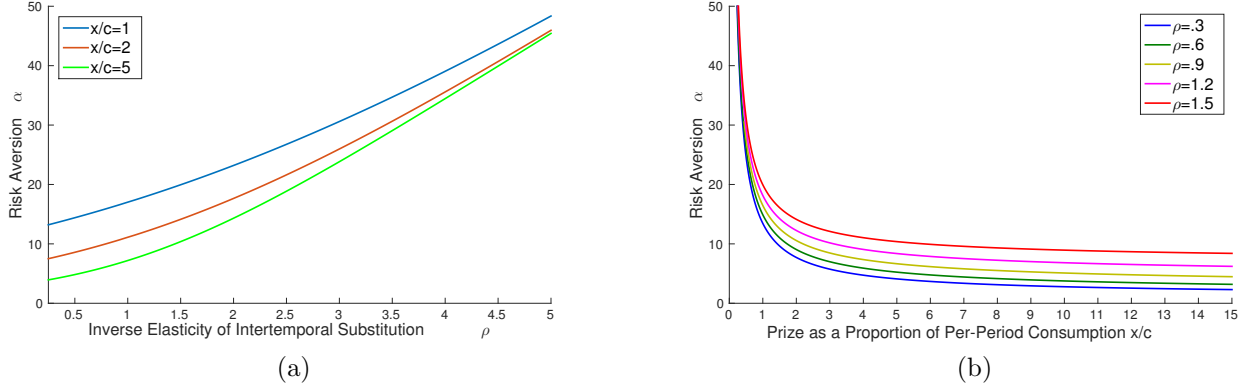
An alternative way to generalize DEU is to allow for probability distortions. Consider a decision maker who evaluates a time lottery that pays prize $x > 0$ at t_0 with probability α and at $t_1 \geq t_0$ with probability $1 - \alpha$ according to

$$V_{DPWU}(\langle \alpha, (x, t_0); (1 - \alpha), (x, t_1) \rangle) = \pi(\alpha) V_{DEU}(\delta_{(x, t_0)}) + (1 - \pi(\alpha)) V_{DEU}(\delta_{(x, t_1)}),$$

where $\pi : [0, 1] \rightarrow [0, 1]$ is increasing and continuous. This formula generalizes many non-Expected Utility models such as rank dependent utility (Quiggin, 1982; Yaari, 1987), cumulative prospect theory (Tversky and Kahneman, 1992), and disappointment aversion (Gul, 1991).¹⁰ For concreteness, we refer to any preferences satisfying this condition as Discounted Probability

¹⁰In the context of static choice under risk and uncertainty, Ghirardato and Marinacci (2001) studied the general class of preferences with this property, which they termed biseparable.

Figure 3: The function $\bar{\alpha}_{\rho,\beta,x}$ determines whether the risky lottery is preferred to the safe lottery



Notes. Each panel depicts parameters under which a decision maker with discount parameter $\beta = 0.9$ will be indifferent between the safe lottery $\delta_{(x,3)}$ and the risky lottery $\langle 0.5, (x, 2); 0.5, (x, 4) \rangle$, where points above each curve correspond to parameters for which the safe lottery is preferred. Panel (a) has ρ on the horizontal axis and each curve corresponds to the prize x as a proportion of per-period consumption c . Panel (b) has the prize as a proportion of per-period consumption in the horizontal axis. Notice that the safe lottery is preferred when α , ρ , and x/c are large. As x/c decreases to zero, the decision maker eventually prefers the risky lottery.

Weighting Utility (DPWU).

Consider again a choice between the safe lottery $\delta_{(x,t)}$ and the risky lottery that pays either $(x, t-1)$ or $(x, t+1)$ with equal probabilities. We now show that within DPWU, the more convex the discount function is, the more likely that the risky lottery is preferred; and the more probabilities are underweighted (that is, the lower $\pi(\alpha)$ is for any α), the more likely that the safe lottery is preferred. To make this statement precise, we need a measure of the convexity of the discount function. Let

$$d_t \equiv \frac{D(t-1) - D(t)}{D(t) - D(t+1)}$$

denote the degree of convexity at t . In particular with exponential discounting we have $d_t = \frac{1}{\beta}$, so a lower discount parameter implies a more convex discount function.

Straightforward calculations show that the safe time lottery is preferred if and only if

$$D(t) \geq \pi(0.5) D(t-1) + (1 - \pi(0.5)) D(t+1),$$

which can be rearranged as

$$d_t \leq \frac{1}{\pi(0.5)} - 1. \quad (7)$$

Thus, the safe time lottery is chosen by individuals with less convex discount functions and those who underweight probability more, showing that DPWU can accommodate local RATL at any t . Moreover, unlike with EZ, the condition for the safe lottery to be chosen (7) does not depend

on the prize x .

For example, with exponential discounting ($D(t) = \beta^t$), the decision maker prefers the safe lottery if and only if $\beta \in (\sqrt{\frac{\pi(0.5)}{1-\pi(0.5)}}, 1)$. Thus, if the decision maker distorts probabilities pessimistically – leading to $\pi(0.5) < 0.5$ and thus $\frac{\pi(0.5)}{1-\pi(0.5)} < 1$ – and if β is high enough – the discount function is not too convex – then she will choose the safe lottery.

The following proposition summarizes the result from above and establishes that DPWU cannot accommodate preferences that are locally risk averse at any x (and, therefore, cannot be globally RATL).

Proposition 4. *Under DPWU, preferences are locally RATL at time t if and only if (7) holds. Moreover, they cannot be locally RATL at any prize x .*

Proof. See Appendix A. □

4 Experiment

4.1 Design

We conducted the experiment at the Wharton Behavioral Lab at the Wharton School of the University of Pennsylvania in October and November 2013. We ran 16 sessions with 197 subjects, recruited from a database of volunteer students. Each session lasted about 45 minutes and no subject participated in more than one session. The experiment was conducted on paper-and-pencil questionnaire. No time limit was given to answer each question. Average earnings were \$25.20, including a \$10 show-up fee. Appendix D includes the complete questionnaire and instructions.

Some of the questions in the experiment involved payments to be made on the same day. If selected for payment, those would be paid at the end of each session, along with the show-up fee. Other questions involved payments to be made in the future. For these, subjects were told that their payment would be available to pick up from the lab starting from the date indicated. All payment dates were expressed in weeks, with the goal of reducing heterogeneity in transaction costs between the dates, under the assumption that students have a regular schedule each week during the semester.¹¹

¹¹An email was then sent to remind them of the approaching date (and they were told they would receive it). Subjects were also given the contact details of one of the authors, Daniel Gottlieb (at the time a full-time faculty at Wharton), in case they had questions about payments. Returning to the lab to collect the payment involve transaction costs, a typical concern for experiments involving delayed payments. However, transaction costs are less concerning for our experiment where *all* payments related to time lotteries take place in future dates. A second concern is that our questions involve payments of different amounts of money over time, and it is well-known that using money to study time-preferences may be problematic (Augenblick et al., forthcoming). However, for the purpose of our study most of these concerns do not apply (for example, the curvature of the utility function is inconsequential on ranking of time lotteries, as we have seen). Moreover, an important part

We ran two treatments: ‘long delay’ and ‘short delay’ (labeled Long and Short). A total of 91 and 105 subjects participated in each, respectively. Both treatments had identical questionnaires except for the length of delays in some of the questions: in the Long treatment, some payments were delayed by up to 12 weeks; in the Short treatment, the maximum delay was 5 weeks. Testing both treatments allows us to study long times spans, where differences between time lotteries become more pronounced; as well as shorter ones, where students’ schedules are more stable, reducing heterogeneous sources of variation.¹²

The experiment has three parts.¹³ Subjects received general instructions about the experiment and specific instructions about the first part of the experiment when they entered the room. Separate instructions were distributed in the beginning of each of the following parts.

Part I asks subjects to choose between different time lotteries and is the key part of our experiment. For example, the first question asked them to choose between \$15 in 2 weeks or \$15 in 1 week with probability .75 and in 5 weeks with probability .25. Subjects answered five questions of this kind. Table 1 lists the questions asked in each treatment. All questions offered two options that paid the same prize at different dates, and one distribution of payment dates was a mean preserving spread of the other. Hence, these questions allow us to elicit subjects’ attitudes towards time lotteries. In three of them, one of the options had a known date; in the other two, both options had random payment dates. All subjects received the same first question (Question 1 in Table 1) in a separate sheet of paper. The answer to this question is a key indication of the subjects’ preferences, as it captures their immediate reaction to this choice, uncontaminated by other questions.¹⁴

Parts II and III use the multiple price list (MPL) method of Holt and Laury (2002) to measure time and risk preferences. With MPL, each question has a table with two columns and twenty-one rows. One column always displays the same option – e.g., receiving \$10 today – while each row in the right column offers a slightly better option in constant increments. For example, in the first question, the options on the right go from \$10 in 2 weeks to \$15 in 2 weeks with .25c increments per row. For each row, subjects have to choose between the left and the right options. These questions are typically interpreted as follows: if a subject chooses the option on the left for all rows above a point, and the option on the right below that point, then the indifference point should be where the switching takes place. Subjects who understand the

of our analysis is the relationship between risk aversion over time lotteries and atemporal risk aversion. Since atemporal risk aversion is only defined for monetary lotteries, we focus our experiment on lotteries over money and leave for future research an investigation of time lotteries involving different objects.

¹²In the Short version all payments were scheduled before the end of the semester in which the experiment was run, and no payment was scheduled during exam week.

¹³The order of parts and of questions in each part were partly randomized: we first describe each part, and then discuss the randomization procedure.

¹⁴One potential concern with offering a list of similar questions is that subjects may ‘try’ different answers even if they have a mild preference in one direction with some hedging concern in mind (Agranov and Ortoleva, 2015).

Table 1: Questions in Part I

Long Delay					Short Delay				
Q.	\$\$	Option 1	<i>vs.</i>	Option 2	\$\$	Option 1	<i>vs.</i>	Option 2	
1	\$20	2 wk		75% 1 wk, 25% 5 wk	\$20	2 wk		75% 1 wk, 25% 5 wk	
2	\$15	3 wk		90% 2 wk, 10% 12 wk	\$15	3 wk		50% 1 wk, 50% 5 wk	
3	\$10	2 wk		50% 1 wk, 50% 3 wk	\$10	2 wk		50% 1 wk, 50% 3 wk	
4	\$20	50% 2 wk, 50% 3 wk		50% 1 wk, 50% 4 wk	\$20	50% 2 wk, 50% 3 wk		75% 2 wk, 25% 4 wk	
5	\$15	50% 2 wk, 50% 5 wk		75% 1 wk, 25% 11 wk	\$10	50% 2 wk, 50% 5 wk		75% 3 wk, 25% 5 wk	

Notes. Each lottery pays the same prize with different delays (in weeks). Subjects in the long delay treatment chose between ‘Option 1’ and ‘Option 2, Long Delay.’ Those in the short delay treatment chose between ‘Option 1’ and ‘Option 2, Short Delay.’

procedure should not switch more than once.

Part II measures time preferences and attitudes towards time lotteries (see Table 2). Questions 6-9 measure discounting between various dates, allowing us also to quantify its convexity. Questions 10 and 11 allow us to quantify risk preferences towards time lotteries. Part III measures atemporal risk preferences, with payments taking place immediately at the end of the session (see Table 3). From these questions, we can quantify each subject’s (standard) risk aversion. Moreover, by asking Allais’ common-ratio-type questions (see for example Questions 12 and 13), we can determine which subjects behave according to Expected Utility theory and quantify violations of it. Finally, at the end of the experiment subjects answered a non-incentivized questionnaire.

Table 2: Questions in Part II

Long Delay				Short Delay		
Q.	Option 1	<i>vs.</i>	Option 2	Option 1	<i>vs.</i>	Option 2
6	\$10 today		x in 2 wk	\$10 today		x in 2 wk
7	\$10 in 1 wk		x in 2 wk	\$10 in 1 wk		x in 2 wk
8	\$10 in 1 wk		x in 5 wk	\$10 in 1 wk		x in 3 wk
9	\$10 in 1 wk		x in 12 wk	\$10 in 1 wk		x in 4 wk
10	\$20 in 4 wk		\$20, $x\%$ in 2wk, $(1-x)\%$ in 12wk	\$25 in 3 wk		\$25, $x\%$ in 2wk, $(1-x)\%$ in 5wk
11	\$25 in 2 wk		\$25, $x\%$ in 1wk, $(1-x)\%$ in 5wk	\$25 in 2 wk		\$25, $x\%$ in 1wk, $(1-x)\%$ in 5wk

Notes. Questions 6-9 ask the amount $\$x$ that would make subjects indifferent between each option. Questions 10-11 ask the probability $x\%$ that would make subjects indifferent between each option. These amounts were determined using MPL.

After all subjects completed the questionnaire, one question was randomly selected from Parts I, II, and III for payment. The randomization of the question selected for payment, as well as the outcome of any lottery (if the selected question had random payments), was made with

Table 3: Questions in Part III

Q.	Option 1	<i>vs.</i>	Option 2
12	\$15		$x\%$ of \$20, $(1-x)\%$ of \$8
13	50% of \$15, 50% of \$8		$x\%$ of \$20, $(1-x)\%$ of \$8
14	20% of \$15, 80% of \$8		$x\%$ of \$20, $(1-x)\%$ of \$8
15	\$20		$x\%$ of \$30, $(1-x)\%$ of \$5
16	50% of \$20, 50% of \$5		$x\%$ of \$30, $(1-x)\%$ of \$3
17	10% of \$20, 90% of \$5		$x\%$ of \$30, $(1-x)\%$ of \$3

Notes. Questions ask the probability $x\%$ that would make subjects indifferent between each option, determined using MPL. All payments were scheduled for the day of the experiment.

the use of dice.¹⁵ Crucially, all uncertainty was resolved at the end of the experiment, including the one regarding payment dates. The instructions explicitly stated that subjects would know all payment dates before leaving the room.

The order of parts and of questions within parts was partly randomized. All randomizations took place at a session level. Therefore, all subjects in the same session saw the same questionnaire, but different questionnaires were used in different sessions. Because Part I is the key part of the experiment, all subjects saw it first to avoid contamination. For the same reason, within Part I, Question 1 was always the same. The other elements were randomized.¹⁶ We find no significant effects of ordering.¹⁷

We conclude with a short discussion of our incentive scheme. The random payment mechanism, as well as the multiple price list method, are incentive compatible for Expected Utility maximizers, but not necessarily for more general preferences over risk.¹⁸ Since this is the procedure used by most studies, a significant methodological work has been done to examine whether

¹⁵Specifically, at the end of the experiment one participant was selected as ‘the assistant,’ using the roll of a die by the experimenter. This subject was then in charge of rolling the die and checking the outcome to determine payments. This was done to reduce the fear that the experimenter could manipulate the outcome. All was clearly explained in the initial instructions.

¹⁶Specifically: for questions in Part I other than the first, half of the subjects answered questions in one specific order (the one used above), while the other half used a randomized order. In each of them, which option would appear on the left and which on the right was also determined randomly. The order of Parts II and III was randomized. For both parts, it was determined randomly whether in the MPL the constant option would appear on the left or on the right. This was done (independently) for each part, but not for each question within a part: in Part II or III the constant option of the MPL was either on the left or on the right for all questions of that part. This is typical for experiments that use the MPL method, as it makes the procedure easier to explain.

¹⁷The only exception is that out of the five questions in the first part, subjects have a significant (moderate) preference for the option on the right in the second question. While this is most likely a spurious significance (due to the large number of tests run), the order was randomized for all sessions and thus this tendency should have no impact on our analysis.

¹⁸Holt (1986) points out that a subject who obeys the reduction of compound lotteries but violates the independence axiom may make different choices under a randomly incentivized elicitation procedure than he would make in each choice in isolation. Conversely, if the decision maker treats compound lotteries by first assessing the certainty equivalents of all first stage lotteries and then plugging these numbers into a second stage lottery (as in Segal, 1990), then this procedure is incentive compatible. Karni and Safra (1987) prove the non-existence of an incentive compatible mechanism for general non-Expected Utility preferences.

this creates relevant differences, with some reassuring results.¹⁹

4.2 Results

We start with two preliminary results. First, most subjects (82%) exhibit convex discounting. Second, the large majority of subjects gave monotone answers to MPL questions: 13% gave a non-monotone answer in at least one of the 12 MPL questions, and only 4.6% gave non-monotone answers in more than one. These are substantially lower numbers (i.e., fewer violations) than what previous studies have found (Holt and Laury, 2002).²⁰

4.2.1 Risk Aversion over Time Lotteries

Our main variable of interest is each subject’s risk attitude towards time lotteries, which can be measured in three different ways. First, we can measure it using Question 1 of Part I, the first question that subjects see. Second, we can look at the answers to all five questions in Part I and ask whether subjects exhibited RATL in the majority of them (for the purpose of this section, we say that subjects are RATL in a given question if, in that question, they chose the option with the smallest variance of the payment date). A third way is to look at the answers given in Questions 10 and 11 of Part II, using MPL.

Table 4 presents the percentage of RATL answers for each of these measures. The results are consistent: in most questions, especially in the Long treatment, the majority of subjects are RATL. In Question 1, the proportions are about 66% for the Long treatment and 56% for the Short one. Notice that subjects are still RATL when *both* options are risky but one of the options is a mean preserving spread of the other (Questions 4 and 5). Thus, the data suggest an aversion to mean preserving spreads, not simply an attraction towards certainty.²¹ In addition, the majority of subjects are RATL in at least one of the two MPL questions (Questions 10 and 11).

In most questions, RATL is stronger in the Long rather than in the Short treatment.²² This is

¹⁹Beattie and Loomes (1997), Cubitt et al. (1998) and Hey and Lee (2005) all compare the behavior of subjects in randomly incentivized treatments to those that answer just one choice, and find little difference. Also encouragingly, Kurata et al. (2009) compare the behavior of subjects that do and do not violate Expected Utility in the Becker-DeGroot-Marschak procedure (which is strategically equivalent to MPL) and find no difference. On the other hand, Freeman et al. (2015) find that subjects tend to choose the riskier lottery more often in choices from lists than in pairwise choices.

²⁰The non-monotone behavior did not concentrate in any specific question. Following the typical approach in the literature, these answers are disregarded. Alternatively, we could have dropped any subject that exhibits a non-monotone behavior at least once. Doing so does leave our results essentially unchanged (as to be expected given the number of them).

²¹We also note that we find no gender differences in RATL, or relationship with results in the SATMath or the total SAT.

²²In Question 10, subjects appear to be more RSTL in the Long than in the Short treatment. This could reflect a genuine preference, or it could be because of the specifics of the MPL for this question: to be RATL in the Long treatment, a subject would have to ‘switch’ very close to the end of the list (row 18 out of 21). It is

Table 4: Percentage of RATL in each question

Question	Long	Short
1	65.71	56.04
2	50.48	54.95
3	48.57	37.36
4	64.76	38.46
5	73.33	52.75
Majority in 1-5	64.76	49.45
10	44.23	54.44
11	57.28	41.11
Either in 10 or 11	64.07	66.66

Table 5: Frequency of RATL answers in Part I

Frequency of RATL	Long Delay		Short Delay	
	Percent	Cum.	Percent	Cum.
0	2.86	2.86	9.89	9.89
1	9.52	12.38	16.48	26.37
2	22.86	35.24	24.28	50.55
3	23.81	59.05	26.37	76.92
4	28.57	87.62	19.78	96.70
5	12.38	100.00	3.30	100

intuitive: when the time horizon is relatively short, the difference between the options decreases and subjects should become closer to being indifferent – and their choices closer to an even split. In the Long treatment the difference in time horizon increases, and so does the differences between the options. While the standard model suggests that this should push more strongly towards RSTL, the *opposite* holds in our data.

While most answers are consistent with RATL, it could be that a non-trivial fraction of our subjects still consistently chooses the risky option, as predicted by DEU. However, Table 5 shows that the fraction of subjects who does so is minuscule in the Long treatment (2.86%) and very small in the Short one (9.89 %). By contrast, in the Long treatment almost 41% give risk averse answers at least 4 out of 5 times, and 59% at least three times. (These numbers are about 23% and 48.45% in the short treatment.)

Result 1. *The majority of subjects is RATL. This is more pronounced in the Long rather than Short treatment. Only a very small fraction is consistently RSTL.*

4.2.2 RATL and convexity, probability weighting, and atemporal risk aversion

In this subsection we analyze the relationship between RATL and convex discounting, violations of Expected Utility, and atemporal risk aversion. With DEU, all subjects with convex discounting should be RSTL; in turn, this means that such tendency should be negatively related to convexity of the discount function. With EZ, RATL should be positively correlated with atemporal risk aversion. With DPWU, only subjects who underweight probabilities should exhibit RATL.

We test these predictions using our data. We quantify convexity of the discount function, violations of Expected Utility, and atemporal risk aversion using the MPL measures collected

well-known that with MPL subjects tend to switch close to the middle of the table, generating a bias towards RSTL in this case. (In all other cases, the switching point exhibited by an RATL subject was after the middle but closer to it.)

in Parts II and III. We first analyze RATL in the restricted samples of subjects with convex discounting, those who behave according to Expected Utility, and those who do not underweight probabilities. We then move to the regression analysis.

As previously described, we determine which subjects have convex discounting based on their answers to Questions 7, 8, and 9 in Part II.

There are two related measures of violations of Expected Utility, constructed using questions from Part III. The first measure uses answers from Questions 12 and 13, or 12 and 14 (see Table 3) to determine if subjects display what is typically called certainty bias (Kahneman and Tversky, 1979).²³ This is the relevant measure for our analysis since subjects who underweight probabilities must also display certainty bias. We find that a small number of subjects do (15.71%).²⁴

The second measure uses the answers to Questions 12, 13, and 14 to determine whether they are jointly consistent with Expected Utility. Since this is a very demanding requirement (it is well-known that these measures can be noisy), we consider as “approximately Expected Utility” those subjects who abide by Expected Utility in all three questions, allowing for a “one-line mistake” – more precisely, they would be consistent with Expected Utility if we changed their answer to these questions by one line. These are 39.89% of the pool (43.3% and 36.67% in the Long and Short treatments, respectively).

Table 6 shows that, based on the four different measures, subjects are still RATL in each of the subsamples described above. The table also shows the results of Chi-squared tests on whether subjects in each of subsample are statistically different from those outside of it. Using Question 1, subjects in any of these subsamples are not statistically different from those in their complements at the 5% level. Moreover, in all treatments and for all RATL measures, certainty-biased subjects are statistically indistinguishable from those with no certainty bias at the 5% level. Similarly, approximately Expected Utility subjects were statistically different from the rest of the population only for the Short treatment and only for Question 10 – where approximately Expected Utility subjects tend to be *more* RATL (the opposite of the prediction of DPWU). Subjects with convex discounting were only statistically different for the Short treatments in the

²³Suppose that in Question 12 the subject switches at x_{12} , while in Question 13 she switches at x_{13} . If the subject follows Expected Utility, we should have $2x_{13} = x_{12}$. A certainty-biased subject would instead have $x_{12} > 2x_{13}$; because she is attracted by the certainty of Option 1 in Question 12, she demands a high probability of receiving the high prize in Option 2 to be indifferent. Thus, the answers to Question 12 and 13 allow us to identify subjects who are certainty biased and to quantify the bias (by $x_{12} - 2x_{13}$). A similar measure can be obtained from the answers to Questions 12 and 14. In what follows, when we want to focus on subjects who are certainty biased, we focus on the measure obtained from Questions 12 and 13 (the results using the measure obtained from Questions 12 and 14 are essentially identical and are reported in Appendix C). When we need to quantify certainty bias (in the regression analysis), we use instead the principal component of the two measures, which should reduce the observation error (essentially identical results hold using either of the two measures or their average.)

²⁴These small numbers are not surprising: it is a stylized fact that certainty bias is less frequent when stakes are small, as in our experiment (Conlisk, 1989; Camerer, 1989; Burke et al., 1996; Fan, 2002; Huck and Müller, 2012). See the discussion in Cerreia-Vioglio et al. (2015).

Table 6: Proportion of RATL subjects

Sample Treatment	Convex Discounting		Approximately Expected Utility		No Certainty Bias	
	Long	Short	Long	Short	Long	Short
Question 1	67.78	50.70*	66.67	60.61	67.50	55.00
Majority in Q1-5	65.56	43.66**	64.29	60.61*	68.75*	50.00
Question 10	46.07	52.86	54.76*	68.75**	47.50	51.90
Question 11	57.95	44.29**	64.29	56.25	54.43	48.10
Observations	90	71	42	33	80	80

Notes. The first row measures RATL using Question 1. The second row identifies as RATL subjects who chose the safe option in the majority of Questions 1-5. The third and fourth rows use answers to MPL Questions 10 and 11. Columns present the proportion of RATL subjects in the subsamples of subjects with convex discounting, approximately Expected Utility, and those with no Certainty Bias as measured using Questions 12 and 13. * and ** denote significance at the 10% and 5% level in a Chi-squared test of whether each subset is different from its complement.

Majority of Questions 1-5 and in Question 11.²⁵ These results are in direct contrast with the predictions of DEU and probability weighting models: according to the former, there should be no RATL with convex discounting; according to the latter, there should be no RATL without certainty bias, or within approximately Expected Utility subjects.

A regression analysis confirms these results. Table 7 presents the coefficients of Probit regressions of RATL with our measures of certainty bias and of convexity as independent variables. (Essentially identical results hold when considering each in isolation; see Appendix C.) Two patterns emerge. First, certainty bias is generally not related to RATL. With the exception of the Short treatment in Question 10 (regression 6), certainty bias is either statistically insignificant, even at the 10% level, or its sign is the opposite of what theory predicts. Second, there is only a significant relationship between convexity and RATL for the Short treatment, and only for the RATL measures using the majority of Questions 1-5 or Question 11 (regressions 4 and 8). In all other regressions, convexity is either insignificant at the 10% level, or it has the opposite sign relative to what theory predicts. Both patterns contradict the predictions of DEU and DPWU.

Lastly, we examine the relationship between RATL and atemporal risk aversion. Table 8 presents the coefficients from a Probit regression, with our four RATL measures as dependent variables and the degree of risk aversion (as measured in Question 12) as the independent variable. Consistently with EZ, the coefficients are positive and, with the exception of the Short treatment in Question 1, they were all statistically significant at the 5% level. In fact, all treatments in Questions 10 and 11 were statistically significant at the 1% level. Similar results hold constructing risk aversion from Question 15 or using a linear probability model.

²⁵Even this should be taken with caution: since we are simultaneously running 24 tests, this can easily be spurious.

Table 7: Probit Regressions: RATL and Convexity and Certainty Bias

Dep. Variable	RATL Q1		RATL Majority Q1-5		RATL Q10		RATL Q11	
Treatment	Long	Short	Long	Short	Long	Short	Long	Short
(Probit)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Certainty Bias	-.25*	.18	-.20	.18	-.03	.44***	.16	.23
	(-1.94)	(1.17)	(-1.60)	(1.20)	(-.25)	(2.77)	(1.42)	(1.52)
Convexity	4.27*	-4.45	.06	-11.10***	3.47	-1.77	.99	-7.73**
	(1.82)	(-1.29)	(.03)	(-2.86)	(1.64)	(-1.29)	(.46)	(-2.16)
Constant	.19	.28*	.39**	.19	-.26	.16	.19	.12
	(1.07)	(1.82)	(2.15)	(1.20)	(-1.52)	(1.03)	(1.08)	(.79)
Pseudo- R^2	.06	.02	.02	.01	.02	.07	.02	.06
Obs.	92	86	92	86	92	85	92	85

Notes. Dependent variables are indicated in the first row. Coefficients in brackets are z-statistics. *, **, and *** denote significance at the 10%, 5% and 1% level.

Table 8: Probit Regressions: RATL and Atemporal Risk Aversion

Dep. Variable	RATL Q.1		RATL Majority Q.1-5		RATL Q.10		RATL Q.11	
Treatment	Long	Short	Long	Short	Long	Short	Long	Short
(Probit)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Risk Aversion,	.336**	.175	.308**	.341**	.459***	.435***	.571***	.239***
Atemporal	(2.41)	(1.30)	(2.27)	(2.40)	(3.27)	(2.91)	(3.81)	(1.75)
Constant	.07	-.01	.07	-.34*	-.60***	-.27	-.37***	-.19
	(0.38)	(-0.07)	(0.38)	(-1.79)	(-3.01)	(-1.47)	(-1.87)	(-1.08)
Pseudo- R^2	0.047	0.014	0.040	0.049	0.083	0.076	0.121	0.025
Observations	101	90	101	90	101	89	100	89

Notes. Dependent variables are indicated in the first row. Atemporal risk aversion measure is obtained from Question 12. RATL measures were obtained from Question 1 (Regressions 1 and 2), having chosen the safe option in the majority of Questions 1-5 (Regressions 3 and 4), and MPL Questions 10 and 11 (Regressions 5-8). Coefficients in brackets are z-statistics. *, **, and *** denote significance at the 10%, 5% and 1% level.

Result 2. *Subjects who exhibit convex discounting, no certainty bias, or are approximately Expected Utility also have a tendency to be RATL. In fact, the proportions in these groups are almost identical to the one in the overall population. Regression analysis shows that RATL is unrelated to probability distortion and generally unrelated to convexity. It is, however, related to (atemporal) risk aversion.*

These findings are not compatible with Discounted Expected Utility or with Discounted Probability Weighting Utility. Instead, they are compatible with Epstein Zin.

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