School Competition and Product Differentiation*

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Abstract

This paper explores how market structure affects private schools' choices of quality in response to competition when quality has a match-specific component. I develop and estimate an equilibrium model of school competition in Pakistan, a country with a large private schooling market. The estimates show that match-specific quality matters: moving a student from her worst to best possible match school increases test scores by 0.3 s.d. Profit-maximizing private schools choose their match-specific quality in response to the marginal rather than the average student. Since rich students are more responsive to quality when they make enrollment decisions, the average private school chooses a match-specific quality that advantages rich students at the expense of poor students. From the point of view of maximizing learning, match-specific quality is significantly misallocated. The entry of an additional private school exacerbates the incentive to cater to wealthy students, increasing inequality *within* private schools by 0.1 s.d.

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1 Introduction

A student who knows calculus will not benefit from remedial math instruction, no matter how well taught. Similarly a student who has never learned fractions is unlikely to benefit from calculus. A growing literature shows that the match between a student's instructional needs and a school's instructional level is an important determinant of learning (Arcidiacono et al., 2016; Arcidiacono et al., 2011; Aucejo, 2011; Duflo et al., 2011; Muralidharan et al., 2016; Kremer et al., 2013). Given the importance of this match, instructional level may be misallocated, with students attending schools whose instruction is not well-matched to their needs, reducing learning. Yet little is known about the extent of this misallocation, how schools choose their instructional level, and how their choices of instructional level respond to competition.

To answer these questions, I develop a novel structural model of how private schools choose their instructional level strategically in response to competitive incentives. In so doing, I draw on insights from the literature in industrial organization focusing on how market structure affects what products firms offer.[1] I estimate this model using data from Pakistan's rural private schooling market. This is a particularly relevant context for understanding the effects of private school competition since private school enrollment is large and fast-growing in Pakistan, as is the case in many low income countries.[2] Given the extremely low rates of learning in these countries[3] (Pritchett, 2013) and the increasing popularity of policies that incentivize school competition,[4] understanding how instructional match interacts with private schools' competitive incentives may be crucial for improving students' outcomes. Moreover, the mismatch between schools' instructional levels and students' needs is likely to be especially important in low income countries, where the typical primary school has one or fewer teachers per grade, making it difficult for schools to assign students to classrooms based on their ability.

To motivate my structural model, I present reduced-form evidence that increased competition increases inequality in learning in Pakistan's private sector. Exploiting variation in the number of private schools in a village caused by the exit and entry of private schools over

[1]Examples of this literature include Mazzeo (2002), Fan (2013), Crawford and Shum (2006), Draganska et al. (2009), Wollmann (2016), and Crawford and Yurukoglu (2012).

[2]In the context that I study, rural Pakistan, 35% of students are enrolled in private schools (Andrabi et al., 2006). In rural India, private school enrollment is 28% (Pratham, 2012) and in urban India, it is 65% (Desai et al., 2008). In sub-Saharan Africa, between 10-54% of primary school children were enrolled in private schools in 2012 (World Bank Development Indicators, 2014).

[3]In the context of Pakistan, Das et al. (2006) find that only 31% of third grade students can correctly form a sentence in Urdu (the vernacular) containing the word "school."

[4]For example, in 2009, India passed the Right to Education Act in India, which requires that private schools set aside 25% of their seats for poor students with vouchers. Similarly, Colombia, Chile, and Pakistan have all experimented with large-scale voucher programs.

time, I find that private sector test scores do not increase with increased competition, but village-level inequality in test scores within the private sector does. Event study graphs show that the increase in inequality coincides with entry events and is not explained by pre-trends. Moreover, this growth in inequality is driven by the growth in inequality in test scores within schools for students who stay in the same school and is robust to including standard controls for time-varying peer effects. These findings suggest that entry leads schools to change their quality in ways that differentially affect different students. In other words, schools change their match-specific quality to benefit students who are already doing well relative to more poorly performing students. To understand why this is the case, we need to understand how entry events affect private schools' incentives.

To test whether schools strategically choosing their match-specific quality can rationalize the increase in inequality, I estimate an equilibrium model of student demand and school supply where students choose to enroll in a school based on its characteristics, and private schools select their quality to maximize their profits. In this model, I allow students' school choice behavior and match-specific school quality to depend on whether the student's wealth is above or below the median for the private school population. I allow for this heterogeneity in particular because wealth is among the strongest determinants of educational outcomes in Pakistan alongside which school a student attends (Das et al., 2006), wealth is unlikely to be causally affected by private school entry, and a growing literature suggests that the school choice behavior of wealthier families differs from that of poorer families throughout the world.[5] I estimate the determinants of students' demand for different schools, allowing poor and rich students' perceived utilities at a school to depend differentially on distance to the school, school fees, and type-specific predicted test score gains. The estimates suggest that poor students are much less responsive to school quality when they make enrollment decisions,[6] in line with the prior literature. This is also consistent with descriptive statistics from my data showing that poor parents are less likely to know their child's teacher's name, are less likely to report choosing a school based on its quality, and have beliefs about school quality that are less correlated with schools' true qualities, as measured by test score gains.

Using the parameters from the demand estimation and the assumption that private schools are profit-maximizing, I can then identify each private schools' equilibrium choice of match-specific quality. I model a school's instructional level as its location on a Hotelling

[5]Bayer et al. (2007) show that higher income households have a higher willingness to pay to live in neighborhoods where schools have higher average test scores, and Ajayi (2011) and Hoxby and Avery (2012) show that poor students make less sophisticated decisions when they choose schools. Relatedly, Dizon-Ross (2015) shows that poor parents have less information about their children's academic performance than rich parents, and Kapor et al. (2017) show that lower income parents' have less accurate beliefs about admissions' probabilities in the New Haven school district.

[6]On average, a school increasing its predicted test score gains for wealthy students will increase its total number of students by six times as many as if it increases its predicted test score gains for poor students.

line, where poor students' optimal instructional level is on the far left of the line and rich students' is on the far right. I find that the average private school chooses an instructional level that strongly favors wealthy students and virtually no private schools choose instructional levels that favor poor students. Moreover, these estimates rationalize the relationship between private school entry and test score inequality: an increase in the number of private schools in the market is associated with the average school moving its instructional level toward rich students at the expense of poor students, reducing learning for poor students and increasing it for rich students in the same school. The estimates from the structural model deliver a more precise prediction: the increase in inequality will be driven mainly by a reduction in poor students' test scores rather than an increase in rich students' test scores. Poor students' learning falls more than rich students' increases because schools cater strongly to rich students even before an entry event, and poor students suffer greater losses as the school moves to the right on the Hotelling line.

These adverse effects of increased competition may seem surprising, but they are consistent with a classic result in industrial organization dating back to Spence (1975). Spence (1975) showed that in the presence of imperfect competition, firms will respond to the marginal consumer rather than inframarginal consumers when choosing their quality. Thus, firms do not typically provide the socially optimal quality.[7] Intuitively, schools realize that rich students are more responsive to quality, as the demand estimates show, and have to be catered to more intensively in order to be retained. The structural estimates show that private school entry exacerbates this effect.

Estimating the supply-side of the model also allows me to estimate to what extent a student's test score gains in a school are due to match-specific quality and to what extent they are due to vertical quality (the element of quality that is the same for all students). Intuitively, the importance of match-specific quality is identified because the profit maximization assumption allows me to estimate a school's equilibrium instructional level given the demand estimates. The correlation between the instructional level and the difference in test score gains between poor and rich students in the same school identifies how important match-specific quality is for students' outcomes. I find that match is important: a school can increase test scores for poor students by as much as one-third of a standard deviation (equivalent to three-fourths of a year of education) by moving from wealthy students' optimal instructional level to poor students' optimal instructional level.[8] Examining the correlation between the outcomes of poor and rich students within a school is not sufficient to measure

[7] Unlike firms, the social planner chooses quality based on all the consumers rather than just the marginal consumer.

[8] This estimate is in line with the effects of randomized interventions that improve the targeting of instructional levels in Kenya (Duflo et al., 2011) and India (Muralidharan et al., 2016).

the importance of match-specific quality in the learning production function. This is because the observed correlation between students' outcomes is affected by both the importance of instructional match for learning and schools' equilibrium choices of instructional level. If all schools choose an instructional level in the middle of the Hotelling line, test score gains for poor and rich students will be the same even if match is a very important determinant of learning. Thus, examining these correlations alone may lead one to underestimate the importance of match-specific quality.[9]

Finally, I again exploit the entry and exit of private schools to test the structural estimates' prediction that the increase in inequality is primarily driven by a decrease in poor students' test scores rather than an increase in rich students'. I find that the entry of an additional private school increases the inequality in yearly test score gains between poor and rich students by 0.1 standard deviations, and this is almost entirely driven by a reduction in poor students' test score gains. Moreover, the entry of an additional private school leads poor students to perform worse on easier questions and rich students to perform better on harder questions, consistent with the idea that entry leads private schools to change their curricula to favor students who benefit from more advanced instruction over students who need instruction at a slower pace.

Overall, this paper makes several contributions. First, this is the first paper – to my knowledge – to quantify the effect of match-specific school quality in the learning production function and show that it is large. Additionally, it characterizes the distribution of private schools' choices of match-specific quality, which are a result of market structure, and shows that private schools choose instructional levels that strongly favor the rich half of the private school population at the expense of the poor half. Coupled with the importance of match-specific quality for learning, this leads to two important sources of misallocation from the perspective of a social planner seeking to maximize learning. First, private schools do not choose instructional levels to maximize the learning of their current students, and second, students do not sort into schools with instructional levels that maximize their learning. In light of recent research documenting the low level of learning in low income countries, interventions that leverage the importance of match-specific quality and reduce the misallocation of instructional level may provide an important tool to improve students' outcomes.[10] Beyond showing that private schools choose instructional levels that favor wealthy students, I show that schools' incentives to do so are intensified by increased competition. These results highlight the importance of measuring within-school differences in school quality, as well as

[9]Additionally, because equilibrium choices of instructional level may differ across markets with different market structures, these correlations could be very different across markets, even if match-specific quality enters the learning production function in the same way in every market.

[10]At least the second form of misallocation might be addressed cheaply and without extensive regulation by supplying parents with better information about schools' match-specific qualities.

cross-school differences, when measuring the effects of school competition on inequality.

Finally, this paper develops a new methodology for structurally estimating schools' equilibrium choices of instructional level. This methodology can be applied to private schools in other markets and could also be adapted to the public sector, as long as we take a stance on the objective function of public schools.[11]

This paper relates to several literatures. First, it contributes to a growing literature that estimates structural models of demand (Carneiro et al. (2016) in Pakistan, Neilson (2013) in Chile, Abdulkadiroğlu et al. (forthcoming) in NYC, Agarwal and Somaini (2014) in Cambridge, Walters (2014) among charter schools in Boston, Bayer et al. (2007) in San Francisco, Hastings et al. (2005) in North Carolina, and Dinerstein and Smith (2014) in New York) and supply (Dinerstein and Smith, 2014) in education. On the demand-side, I contribute to this literature by allowing a school's quality to be different for different students and by allowing schools to choose both match-specific and vertical quality.[12] On the supply-side, I contribute to this literature by modeling school's choices of match-specific quality in response to student demand.

Second, it contributes to a growing literature on the effects of private schooling on students' outcomes in a variety of settings – Neal (1997) in the United States, Andrabi et al. (2010) in Pakistan, Muralidharan and Sundararaman (2015) in India, and Neilson (2013) and Hsieh and Urquiola (2006) in Chile. Researchers working in the same (Andrabi et al., 2010) or similar (Muralidharan and Sundararaman, 2015) South Asian contexts have demonstrated that private schools are more productive than public schools, delivering as great or greater levels of learning at lower costs. I build on this important result by showing how market structure can affect the distribution of learning *within* private schools.

Finally, this paper also adds to the literature on the effects of competition on student outcomes (Hoxby, 2000; Hoxby, 2003; Sass, 2006; Booker et al., 2008; Card et al., 2010; Neilson, 2013; Figlio and Hart, 2014; Jinnai, 2014; Hsieh and Urquiola, 2006; Bettinger, 2005; Bifulco and Ladd, 2006; Rothstein, 2007; Mehta, 2015; Imberman, 2011; MacLeod and Urquiola, 2015; MacLeod and Urquiola, 2013; Bayer and McMillan, 2010). In this paper, I estimate the effects of competition due to school entry and allow competition to have heterogeneous effects on rich and poor students in the same private school.

The rest of the paper is organized as follows. Section 2 discusses the context and the data. Section 3 provides motivating evidence on the relationship between private school entry and test score inequality. Section 4 develops the equilibrium model of student demand and

[11]For example, public schools could be assumed to maximize mean test scores or the probability of passing the requirements imposed by an accountability regime.

[12]Vertical quality is the element of school quality that affects all students in the same way. Only vertical quality matters if the school that maximizes learning for one student also maximizes it for all other students. Match-specific quality allows some schools to be good for some students and bad for others.

school supply, and Section 5 structurally estimates the model. Section 6 uses reduced-form techniques to test the predictions of the structural estimation, and Section 7 concludes.

2 Context and Data

2.1 Context

Pakistan is a natural setting to study the private schooling market in low-income countries. Like many low-income countries, Pakistan has experienced a rapid increase in low-cost, secular private schooling over the past two decades. Pakistani private schools are virtually unregulated and are, for the most part, unsupported by the government. As a result, they offer us a glimpse into what mechanisms may be important in a purely private market for education.

Private schools do not merely cater to the wealthy. Andrabi et al. (2008) show that grassroots, rural private schools are even affordable for day laborers, noting that the average school charges the equivalent of "a dime a day." However, private schools are more expensive than public primary schools, which are free. Private schools can afford to charge so little because they typically spend less per student than public schools, largely because public school teachers earn about five times as much as private school teachers. In rural Pakistan, there is almost no overlap in the public and private school wage distribution (Bau and Das, 2016). Thus, public and private schools do not compete to hire teachers from the same labor market, and unlike private schools, public schools are typically constrained to hire teachers who have completed post-secondary degrees.[13] Because the public and private sector hire from different labor markets and because public teacher hiring is centralized at the district level, changes in the local private school market are unlikely to affect public schools through the market for teachers.

Unlike private schools, public schools face relatively little competitive pressure. Historically, most teachers were hired on permanent contracts and are difficult to fire. School budgets are not determined by the number of students enrolled, and schools face little threat of closure if enrollments drop. Public and private schools also differ in that public schools are single-sex, while most private schools are co-educational.

An advantage of studying private schooling in rural Pakistan is that, at the primary level, villages act as closed educational markets. Villages are typically far apart or separated

[13] Public school teachers typically have more education and more teaching training than private school teachers (Andrabi et al., 2008). However, Bau and Das (2016) show that these characteristics do not correlate with teacher value-added estimates. These results suggest that while public school teachers are more qualified, they are not necessarily more effective.

by natural barriers, and students are very sensitive to distance when they make enrollment decisions (Andrabi et al. (2010), Carneiro et al. (2016)). Therefore, I can consider how competition affects equilibrium school and student outcomes at the market level.

The match between schools and students is likely to be a particularly important determinant of students' learning in rural Pakistan because schools' ability to cater to students' instructional needs is limited. Both public and private schools have less than 1 teacher per grade on average (one teacher teaches 1.11 grades in the median private school), suggesting that within-grade tracking is extremely rare and cross-grade mixing within the same class is quite common. In this context, students requiring many different instructional levels are likely to be present in the same classroom.

Finally, neither public nor private schools in rural Pakistan typically have binding capacity constraints. The majority of schools in 2007 had some form of admissions procedure (97% of private schools and 95% of public schools), which frequently consisted of an oral exam (81% of private schools, 54% of public schools) or the perusal of previous school reports (14% of private schools and 33% of public schools). However, even if a student was deemed "weak" based on this assessment, only 11% of private schools and 3% of public schools said they would refuse admission. Therefore, a student typically attends her first choice school. Given this admissions process, a student's primary school is unlikely to impact whether she continues on to secondary school apart from its effect on her learning. This suggests that, in contrast to college selection (MacLeod et al., forthcoming), students are unlikely to choose schools based on reputation effects alone.

2.2 Data

For this paper, I use the Learning and Educational Achievement in Pakistan Schools study (LEAPS). The LEAPS data consists of four rounds of data collected between 2004 and 2007 in a stratified random sample of 112 rural villages in the Attock, Rahim Yar Khan, and Faisalabad districts of Punjab. To be included in the sample, villages were required to have at least one private school in 2003. Therefore, the sampled villages are somewhat more populous and wealthier on average than the average village in the province. The LEAPS data allow for the construction of two partially overlapping samples containing data on household wealth: a sample of tested students surveyed in schools, which I refer to as the *tested sample*, and a sample of children whose parents were surveyed in their geocoded homes, which I refer to as the *household sample*.

Both these samples are necessary for the estimation of the structural model. The sample of tested students allows for the measurement of school quality, a key determinant of students' enrollment decisions, in terms of test score gains. The geocoded household sample allows for the estimation of the determinants of school choice behavior in a sample for which the distance between every school and every student is known. Since distance is a major determinant of school choice and the characteristics of students' closer schools may differ systematically by their wealth, allowing distance to affect students' enrollment decisions is key for taking into account systematic differences in poor and wealthy families' choice sets.

Surveyors visited the universe of schools within a 15 minute walk of the village and collected geocoded data. The initial sample included 823 schools in the first round (2004), and additional schools were added to the sample as they entered the market. In the first round of data collection, all third graders in a school were tested using low-stakes tests administered by the surveyors in math, Urdu, and English. These students were followed and tested in subsequent years, and an additional sample of third graders was added in 2005-2006 and tested subsequently in 2006-2007. In all, the panel includes 71,167 student-year test score observations (31,382 unique students). The sample of tested children comprises the tested sample. Test scores on the exams were calculated using item response theory (see Das and Zajonc (2010) for details), so that the mean test score in the population is 0 and the standard deviation is 1. A random sub-sample of students were also given an additional survey on their household assets, leading to a sample of 28,449 student-year observations for which both test score and survey data is available.

A smaller sub-sample of this tested sample (1,269 households) can be matched to a household survey administered to a panel of 1,740 households in each year. Drawing from a baseline census of the villages, 16 households were sampled in each village. 12 households were randomly chosen among those who had a child attending grade three in the first survey round and 4 were chosen among households where a child eligible to be enrolled in grade 3 was not enrolled.[14] Importantly, in round 1, the location of households was geocoded. As a result, I can compute the geographic distance between a household and the schools in the village for the 7,216 children aged 5-15 who appear in the household survey. This sample comprises the household sample. Households were also asked about their enrollment decisions for each of the children in each year and about their asset ownership, although the asset questionnaire differed slightly from the asset questionnaire administered to the children in the tested sample.

Appendix Table A1 presents summary statistics for the four rounds of LEAPS data at the school level. Because of the panel structure of the data, each observation is at the school-year level. As Appendix Table A1 shows, there is substantial variation across schools in terms of facilities, and private schools generally have better facilities. The average public and private

[14] This matches the fact that 25% of third grade-aged children are not enrolled in school.

schools are mechanically located in villages with more schools. While the average private school is in a village with 4.6 private schools, the average village has 2.5 private schools. Similarly, while the average public school is in a village with 6.0 public schools, the average village has 4.1 public schools. Appendix Table A2 summarizes the characteristics of the children in the tested sample; each observation is at the child-year level. While students in private schools tend to have higher test scores and more assets, there is again substantial variation in student performance and wealth. Finally, Appendix Table A3 reports summary statistics for the household sample, which unlike the tested sample, includes students who are not enrolled.

3 Motivating Evidence on Competition and Inequality

In this section, to motivate the equilibrium model of village educational markets, I study the effects of changes in market structure due to school exit and entry on students' test scores, focusing on students in the private sector. I argue that these exit and entry events, while likely correlated with village characteristics, are plausibly uncorrelated with village or school-level time trends. Importantly, the LEAPs data were collected during a period of rapid private schooling expansion, during which entry was occurring in many markets.[15] During this expansion, private schools may have entered villages that were already large, but not necessarily faster-growing. To examine if this is the case, I use data on village-level populations from the 1998 and 1981 population censuses of Punjab. In columns 1 and 2 of Appendix Table A4, I regress the village populations in 1998 and 1981 on the number of private schools, and I find that population is positively correlated with the number of private schools. However, column 3 shows that the percent change in population between 1981 and 1998 is not significantly correlated with the number of private schools. Thus, entry events did occur in larger villages, but village fixed effects account for the level differences in student outcomes between these villages.

Using the tested sample, I first estimate the effect of private school entry on test scores in both the private and public sectors using the following specification

$$y_{it} = \rho_0 + \rho_1 num\_pri_{vt} + \alpha_t + \psi_v + \lambda_g y_{i,t-1} + \phi_g y_{i,t-1}^2 + \mathbf{\Gamma X_{vt}} + \epsilon_{ivt},$$

where i indexes students, g indexes grades, v indexes villages, and t indexes years. y_{it} is then a student-year level test score in math, Urdu, or English, num_pri_{vt} is the number of private

[15] From 1991-2001, private school enrollment in Punjab increased from 15% to 30%. While data from later years is not readily available, the entry of private schools into the market showed no signs of plateauing by 2000 (Andrabi et al., 2008).

schools in village v at time t, α_t are survey year fixed effects, ω_g are grade fixed effects, ψ_v are village fixed effects, and $\mathbf{X_{vt}}$ is the set of village-year controls.[16] λ_g and ϕ_g are grade specific coefficients on test scores and lagged test scores. Standard errors are clustered at the village-level.[17]

Table 1 reports the results of these regressions for the public and private school populations. The results suggest that the degree of competition in a market does not affect the average student's test scores in either the private or public sector. However, if competition has heterogeneous effects, individual students' outcomes may change even if the average student's test scores remain the same. To see if this is the case, I estimate the effect of exit and entry events on inequality in the private score using the specification:

$$inequality_{vt} = \rho_0 + \rho_1 num\_pri_{vt} + \alpha_t + \psi_v + \mathbf{\Gamma X_{vt}} + \epsilon_{vt},$$

where $inequality_{vt}$ is a village-year level measure of inequality in test scores in the private or public sector (either the variance in test scores or the gap between the test scores of the students at the 90th and 10th percentile). Now, an observation is at the village-year level rather than the student-year level.

Table 2 reports the effect of private school entry on the variance of test scores in the public and private sectors. Outcomes within the public schools again appear to be unaffected by competition. In contrast, the entry of a private school increases the variance in private sector test scores by a statistically significant 0.08 test score standard deviations. Appendix Table A5 investigates the effect of entry on the alternative measure of test score inequality – the gap between the students' test scores at the 90th and 10th percentiles – and again finds that inequality widens with private school entry.

Figure 1 verifies that this result is not driven by pre-existing time trends for both the variance of mean test scores (across Urdu, English and math) and the gap in mean test scores between the students at the 90th and 10th percentiles. The figure plots the γ coefficients and their 90% confidence intervals in the regression

$$\begin{aligned} inequality_{vt} = &\rho_0 + \gamma_1 event_{v,s-2} + \gamma_2 event_{v,s-1} + \gamma_3 event_{v,0} + \gamma_4 event_{v,s+1} + \gamma_5 event_{v,s+2} \\ &+ \alpha_t + \psi_v + \mathbf{\Gamma X_{vt}} + \epsilon_{vt}, \end{aligned} \tag{1}$$

[16]These controls consist of a control for whether the village was treated in a village-level randomized report card intervention, which supplied information on schools' average test scores and student's test scores, studied by Andrabi et al. (2017). I control for this intervention throughout and allow its effects to vary by year.

[17]The choice of specification is motivated by specifications for estimating teacher value-added in Chetty et al. (2014a). Like Chetty et al. (2014a), I control for grade-specific effects of lagged test scores to account for selection into particular schools. In this case, these controls account for the correlation between student selection into a sector and private school entry and exit.

for the private sector, where s denotes the years since an exit or entry event occurred, and the *event* variables takes the value 1 if the event was an entry event, -1 if it was an exit event, and 0 if no event occurred. The event study graph shows that, prior to an entry event, there is no increase in inequality in test scores in villages where entry events occurred and, following the event, there is a significant increase in inequality.

The event study graphs indicate that the growth in inequality is not driven by pre-trends in inequality and test scores. To verify that this is indeed the case, I also estimate the association between private school exit and entry and other village characteristics that may affect inequality in test scores. I first regress a village-year measure of village wealth on num_pri_{vt}, controlling for village and year fixed effects.[18] The coefficient for number of private schools (0.017) is small and statistically insignificant (see column 4 of Appendix Table A4). Using the gini coefficient for wealth as the outcome variable or the percent of households that own land in a village-year yields similar results (columns 5 and 6); the coefficients are again small (0.001 and -0.001 respectively) and insignificant. Thus, there is no strong relationship between the number of private schools in a village and village-level trends in wealth or inequality.

Finally, in Table 3, I investigate the drivers of the increase in test score inequality. To test whether the increase is driven by a change in the composition of the private sector or a change in the composition of the private schools due to entry, I re-estimate equation 1 for the variance in test scores, restricting the sample to private school students who never change schools *and* were observed in a private school prior to the entry event. As the odd columns of Table 3 show, this only strengthens the association between private school entry and inequality. This is unsurprising since school switching in the tested sample is uncommon. Only 23% of students are ever observed changing schools, and the bulk of these changes occur when children finish primary school and move to a middle school.[19]

The even columns of Table 3 investigate two possible drivers of the growth in inequality. In these columns, I recalculate the variance in test scores at the *school-year* level instead of the sector-year level and use this as the outcome variable in equation 1, controlling for school fixed effects instead of village fixed effects. If the growth in learning inequality is driven by a growth in inequality *across* private schools, the number of private schools should not affect this outcome. Additionally, I control for two conventional measures of peer effects – the

[18]To create this measure, I follow Filmer and Pritchett (2001) and predict the first principal component from a principal components analysis over indicator variables for asset ownership for the different assets in the household survey (see Appendix Table A2 for a list of assets).

[19]If I exclude year 4 of the sample, when the first cohort of students is old enough to enter middle school, only 4% of students change schools. Changing between public and private schools is even rarer, with only 2% of students observed in both types of schools. The low rate of switching does not indicate that students do not respond to school characteristics when they choose schools. The switching rate is an equilibrium outcome, and schools may change their characteristics to retain students.

average of the lagged test scores of the students in a school and the variance in the lagged test scores – as well as student-teacher ratios, which vary across years. As the even columns of Table 3 show, I find that the growth in the inequality in test scores is mainly driven by growth in *within-school* inequality, and the peer effects controls do not account for the growth in inequality. As column 8 shows, a new private school increases within-school test score variance by 0.061 test score standard deviations. This specification also suggests that the growth in inequality is driven by the pre-existing schools in the market responding to competition rather than the quality of the new school. Because the specification controls for school fixed effects and entering schools are not observed prior to the entry event, variation in the entering schools' students' test scores associated with the number of schools in the market place is absorbed by the school fixed effects.

The fact that private school entry increases test score inequality in the private sector and that this increase is driven by increasing within-school inequality suggests that match-specific quality matters and that it reacts to market structure. Additionally, the fact that mean test scores in the private sector do not improve suggests that schools are not merely improving with some faster-learning students benefiting more from these improvements than others. Rather, some students' learning is increasing while others' is declining.[20] Motivated by these results, in the next section, I develop an equilibrium structural model to better understand the importance of match-specific quality and test whether strategic incentives can explain these reduced-form findings.

4 Equilibrium Model

In this section, I develop a structural model of students sorting into schools and schools choosing their price and quality to test whether increased competition indeed changes private schools' incentives, leading them to choose match-specific qualities that increase learning for some students and decrease it for others. Based on the evidence in the previous section, I allow schools' effect on students' test scores to have a match-specific, as well as a vertical component. I also allow students to belong to one of two types, depending on their wealth. I choose wealth as my key source of heterogeneity since it is the major, non-school driver of students' learning (Das et al., 2006) and is a well-known determinant of school choice behavior in the literature.[21]

[20]In Section 6, I directly verify that this is the case.

[21]Additionally, Appendix A verifies that wealthy parents are more knowledgeable about the educational market place and self-report placing more weight on school quality when they make enrollment decisions. The model allows students of different types to differ in their responsiveness to school characteristics and their optimal instructional level.

To allow school quality to have both a vertical and match-specific component, I model poor students as having an optimal instructional level of 0 on a unit line, while rich students have an optimal instructional level of 1. School j chooses both its location on the line ($h_j \in [0,1]$) and its vertical quality, v_j. Then, if a school chooses h_j, a poor student's learning (or test score gains) in the school will be

$$VA_{j,poor} = v_j - \beta(o_{poor} - h_j)^2 \tag{2}$$

and a rich student's learning will be

$$VA_{j,rich} = v_j - \beta(o_{rich} - h_j)^2, \tag{3}$$

where o_z is the optimal instructional level of a student of type $z \in \{poor, rich\}$. Here, β weights the importance of horizontal quality relative to vertical quality in the learning production function, and is a parameter of particular interest that I will be able to estimate in the next section. Importantly, the structural model does not assume that match-specific quality matters: only vertical quality matters in the special case where $\beta = 0$.

One limitation of this model is that it does not allow for direct peer effects in the learning production functions in equations 2 and 3. Allowing for direct peer effects substantially complicates the estimation of the model. Since the results in Table 3 show that conventional measures of peer effects do not account for the association between private school entry and inequality that motivate this paper, I choose to abstract away from them.

In the first subsection of this section, I specify student demand for schools as a function of students' types and schools' characteristics, and in the next subsection, I specify the school's problem. In the final subsection, I use a simplified version of the model to develop intuition for why increased competition can lead to greater inequality in learning within private schools.

4.1 Student Decision Problem

A student i of type z chooses the school j that maximizes her perceived utility

$$u_{ijz} = \delta_z VA_{jz} + \mathbf{\Gamma_z} X_{ij} + \xi_j + \epsilon_{ij}, \tag{4}$$

where X_{ij} is the set of other student-school characteristics, such as fees and geographic distance, ξ_j is unobserved school-specific quality, and ϵ_{ij} is an idiosyncratic shock drawn from the type 1 extreme value distribution that may either capture parents' mis-perception of school quality or idiosyncratic taste. $\mathbf{\Gamma_z}$ and δ_z are parameters that govern the effect of school quality and other school characteristics on a student's utility. A student's choice set includes both private and public schools, and a student may also choose the outside option (not enrolling in school), which is normalized to have a utility of zero. If poor students are less responsive to school quality than rich students, as the descriptive statistics in Appendix A suggest, $\delta_{low} < \delta_{high}$.

Since ϵ_{ij} is drawn from the type 1 extreme value distribution, the probability that a student i of type z attends a school j can be written as

$$p_{ijz} = \frac{e^{\delta_z VA_{jz} + \mathbf{\Gamma_z} X_{ij} + \xi_j}}{\sum_k e^{\delta_z VA_{kz} + \mathbf{\Gamma_z} X_{ik} + \xi_k}},$$

where k indexes the other schools in a student's choice set. In practice, a student's choice set consists of the schools in her village and the outside option, but including characteristics like distance in the student's utility allows the attractiveness of different schools to depend on where the student lives within a village.

4.2 School Decision Problem

A private school j chooses its characteristics (including its fee, fee_j, and its vertical quality, v_j) simultaneously with other private schools to maximize its profits. The profits of a private school j are given by

$$\pi_j = fee_j \times s_j(v_j, h_j, fee_j, \mathbf{v}, \mathbf{h}, \mathbf{fee}) - c(v_j, h_j, s_j(v_j, h_j, fee_j, \mathbf{v}, \mathbf{h}, \mathbf{fee})),$$

where c is the school's cost function, $s_j = \sum_i p_{ijz}$ is the number of students attending j, and the bolded variables are vectors of these variables for the other schools in a student's choice set. Differentiating π_j with respect to h_j yields

$$\frac{\partial \pi_j}{\partial h_j^*} = \left(fee_j - \frac{\partial c}{\partial s_j} \right) \frac{\partial s_j}{\partial h_j^*} - \frac{\partial c}{\partial h_j^*} = 0.$$

If we assume that changing a school's instructional level is costless and that c is weakly concave as a function of s_j,[22] for h_j^* to be profit-maximizing, it must be the case that $\frac{\partial s_j}{\partial h_j^*} = 0$. As we will see in Section 5, imposing this assumption makes the problem of identifying and estimating β and h_{jt}^* separable while making only limited assumptions about the shape of c.

In contrast to private schools, I do not make any assumptions about public schools'

[22] The cost, c, will be weakly concave in the number of students if there are constant or increasing economies of scale in providing education.

objective functions. Assuming that private schools are profit-maximizing is sufficient to estimate β and the equilibrium values of h_{jt} for every private school in the market.

4.3 Equilibrium Behavior in a Simplified Model

In equilibrium, private schools choose the vector of characteristics that will maximize their profits and students choose the schools that will maximize their perceived utility. A simplified version of the model is useful to develop intuition for how new entry can incentivize the average private school to choose a h_j closer to 1, increasing learning for wealthy students and decreasing it for poor students. For simplicity, let $\mathbf{\Gamma_z}$ and ξ_j be 0, so that students only respond to horizontal quality, and fix fee_j. Let h_j be the only characteristic that a school can choose. Furthermore, assume there is no outside option besides a passive public school. Thus, a private school j maximizes s_j, and a student i of type z's utility in a school is

$$u_{ijz} = -\delta_z \beta (o_z - h_j)^2 + \epsilon_{ij}.$$

Additionally, assume that δ_{rich} is infinite so that high types always select their best-match option (as in the traditional Hotelling model), while $0 < \delta_{poor} < \infty$.[23] Finally, assume that there are equal numbers of rich and poor types in the market.

To see how competition can affect a private school's incentives, consider the cases with one ($N = 1$) or two ($N = 2$) private schools in the market, and a public school that gives utility $u_0 \leq 0$. The following two propositions characterize schools' equilibrium instructional level when there are one and two schools in the market place, respectively.

Proposition 4.1. *For N=1, there is a unique equilibrium where the single private school chooses* $h^* = \max(1 - (-u_o)^{1/2}, 0)$.

Proof. See Appendix B.

Proposition 4.2. *For N=2, if δ_{low} is sufficiently small, the unique equilibrium is* $(h_1, h_2) = (1, 1)$.

Proof. See Appendix B.

Comparing the equilibrium for $N = 1$ and $N = 2$ shows that the implications of this model are very different from a standard Hotelling model, such as the one developed by Eaton and Lipsey (1975). In a standard Hotelling models, increasing the number of schools

[23] This model, in which students choose their best-match school subject to an idiosyncratic shock, relates to work on the Hotelling model by De Palma et al. (1987) and De Palma et al. (1985), who develop Hotelling models where consumers also have an idiosyncratic shock in their utility function, but in this case, the relative importance of the shock is correlated with a student's location on the Hotelling line.

in the market typically leads to symmetric product differentiation. However, this is no longer the case when there is heterogeneous sorting by rich and poor students.[24]

When there is only one school in the market, it caters more to the poor students because it is more likely to lose them. As long as the rich students prefer the single school to the outside option, the school gains nothing by becoming more attractive to them. On the other hand, it gains poor students continuously as it moves h_1 closer to zero. However, when a second school enters, it changes the first school's incentives, in line with the intuition from Spence (1975). Now, it must compete aggressively to retain the rich students, and since they are more responsive to quality, competing for them has a higher payoff. As a result, in this simplified version of the model, rich students benefit from increasing competition while the poor students' learning falls, increasing inequality. As in Spence (1975), this change in schools' incentives due to competition does not necessarily make the average consumer better off (as measured by learning) because schools choose h_j in response to the marginal consumer rather than the average consumer. While the rich types' learning unambiguously increases, the poor types' decrease in learning can more than cancel out these increases in the average private school. Estimating the more general structural model will allow us to test if this is the case.

5 Estimation of the Equilibrium Model

In this section, I estimate the parameters of the structural model. To do so, I first need to determine each students' type and estimate each types' predicted test score gains in each school. This section is divided into four parts. In the first part, I use students' asset data to determine which students are rich and which are poor. In the second part, combining these estimates of students' types with panel data on students' test scores, I estimate the predicted test score gains of a student of type z in a school j, equivalent to VA_{jz} in the previous section. Using these estimates, in the third subsection, I estimate the parameters of students' demand for schools, given by equation 4 in the previous section. Finally, in the fourth subsection, using the estimates of the parameters of the demand system, I estimate β, the importance of match-specific quality, and schools' equilibrium choices of h_j. With these estimates in hand, I can (1) quantify the importance of the match between instructional level

[24]For simplicity, in this section, I have focused on cases where the pure strategy Nash equilibrium exists. When δ_{low} is not sufficiently low, in the special case of the simplified model where $\delta_{high} = \infty$, the pure strategy Nash equilibrium does not exist for $N = 2$. For a proof of this, see Appendix B. However, in the next section, where I estimate δ_{high} and δ_{low} and then compute the equilibrium for the full model, I do not impose any restrictions on δ_{high} and δ_{low}. In this setting, Schauder's fixed point theorem guarantees that there is an equilibrium in mixed strategies, and when I estimate the model, I estimate each school's equilibrium h_j directly, demonstrating that an equilibrium in h_j exists.

and a students' instructional needs in the learning production function, (2) characterize the distribution of instructional level in the private sector, and (3) test whether h_j moves closer to 1 when there are more private schools in a market, indicating that competition incentivizes schools to choose instructional levels that favor wealthy students.

5.1 Assignment of Rich and Poor Types

It is necessary to assign types to both the students in the data set of children who were surveyed in schools and to the children of parents who were surveyed in their homes. Recall that while these data sets overlap, most children in one data set are not in the other. The panel of tested students is key for estimating schools' quality since it contains data on students test scores over time. The geocoded panel of households is key for estimating the determinants of students' demand for schools since it contains data on the distance between every household and every school.

Neither the LEAPS questionnaire administered to the children tested in schools nor the one administered to parents in their homes directly measures income. Even if the questionnaires did, measures of income are unlikely to measure socioeconomic status well in rural Pakistani villages where the majority of the population is engaged in agriculture. However, the LEAPS survey administered to tested children does ask "yes" or "no" questions about asset ownership for beds, radios, TVs, refrigerators, bicycles, ploughs, small agricultural tools, tables, chairs, fans, tractors, cattle, goats, chickens, watches, motor rickshaws, motorcycles, cars, telephones, and tubewells. To synthesize this data into a single measure that captures the assets that are relevant for educational outcomes, I adapt a simple machine learning technique.

I first regress an indicator variable for ownership of each asset on year and village fixed effects and predict the residual from this regression. This helps ensure that my measure picks up differences in wealth *within villages*. From the perspective of the schools in my model, which compete for students within a village, across-village or across-year differences in wealth are not relevant. I then interact these residual terms to generate double and triple interactions of all the asset ownership measures. Finally, I regress average student test scores across math, Urdu, and English, which have also been residualized, on these 211 covariates using a lasso regression to choose the final specification.[25] Lasso regression is a technique for selecting the most predictive covariates when there are a large number of possible covariates. It selects the covariates that minimize the sum of squared residuals

[25]I use this technique instead of simply counting the number of assets a household owns because the ownership of some assets, like chickens, may be more predictive of belonging to a family that works in agriculture rather than of household wealth and educational outcomes.

subject to an L1 regularization penalty. For more details, see Tibshirani (1996).

I then use a simple rule to determine which students are rich or poor based on their predicted average test scores. I code students as rich if their predicted average test scores are above the median for the private school-going population and as poor otherwise. Therefore, by construction, 50% of private school students in the tested data are poor types. Since private school students are wealthier than the general population, in the full student tested data, 73% of students overall are poor types.[26]

Now, I also need to assign types in the household data, where test scores are not available for most children. The household survey asks about a slightly different set of assets than the tested child survey. For example, the household survey asks about VCR, gun, and thresher ownership, while the tested child survey does not. To generate types for this data set, I again regress the indicator variables for each of these assets on village and year fixed effects and create double and triple-interactions of these residualized asset ownership variables, generating 299 variables. Then, for the 1,269 children who appear in both data sets, I use a logistic version of the lasso regression to select the covariates that are most predictive of their assigned types, resulting in the selection of 14 covariates. I use the estimated coefficients to predict the probability of being a high type for all the children in the household sample. For the children who are assigned both a type from the first procedure and a probability of being a high type from the second procedure, the two measures are strongly related with a correlation of 0.58. To summarize, in the tested sample, children are assigned to their type with certainty, while in the household sample, children are assigned a probability of being each type.

5.2 Estimation of Match-Specific School Quality

To proxy for the match between each type and a particular school (VA_{jz}), I draw on the value-added literature in education economics (for example, see Chetty et al. (2014a), Chetty et al. (2014b), Rivkin et al. (2005), Kane and Staiger (2008), and Rockoff (2004)), to estimate the predicted test score gains of a rich or a poor type attending a given school conditional on the number of competing schools (hereafter referred to as a school's type-specific value-added). In other words, I allow a school's value-added to vary over time with how many private schools are in the village, consistent with the idea that schools will be incentivized to change their instructional level when the number of private schools changes. To calculate a

[26] While, in principle, the model could be generalized to have more than two types, in the next section, I will use the type assignments to estimate each school's type-specific quality. To obtain these (non-parametric) estimates, I must observe sufficiently many students of each type in each school. Therefore, with more and more types, the estimates of type-specific school quality become less credible and these parameters may not be identified.

school's type-specific value-added, using the tested data, I estimate the following regression for math, English, and Urdu:

$$y_{it} = \tau_{gz,1} y_{i,t-1} + \tau_{gz,2} y_{i,t-1}^2 + \omega_{gz} + \alpha_{zt} + \eta_{zjn} + \mathbf{\Gamma X_{vzt}} + \epsilon_{it} \tag{5}$$

where y_{it} is the outcome variable consisting of normalized test scores in math, English, or Urdu, i indexes an individual, g indexes a grade, z indexes a type, j indexes a school, n indexes the number of private primary schools in the market place, and t indexes a year. $\tau_{gz,1}$ is a grade-by-type specific coefficient on the lagged test score, $\tau_{gz,2}$ is a grade-by-type specific coefficient on the lagged test score squared, α_{zt} is a year-by-type fixed effect, ω_{gz} is a grade-by-type fixed effect, η_{zjn} is a fixed effect at the school-type-number of schools in the village level, and $\mathbf{X_{vzt}}$ is a set of village by type by year controls.[27] Then, the type-specific value-added for a type z in a school j under competitive regime n is given by η_{zjn}. To construct a single measure of quality, I average across a school's estimated type-specific value-addeds in math, Urdu, and English to create a type-specific value-added in mean test scores.

The goal of this method is to estimate the causal effect of attending a school on test scores separately for rich and poor students. The controls account for variation in test scores that is explained by year of test-taking, grade of test-taking, and a student's past performance. The remaining unaccounted for variation in test scores is then attributed to the school that students attended. Intuitively, the fixed effect η_{zjn} is the average of the unaccounted for variation in test score gains for different types of students in different schools. Therefore, for these measures to be unbiased, the underlying assumption is that controlling for the lagged test scores and fixed effects accounts for most of the selection of students into schools.

In Appendix Table A6, I test whether these measures are in fact strong predictors of student test scores when students change schools.[28] I show that the type-specific value-added measures are indeed highly predictive of a student's out-of-sample gains from attending a given school. In fact, the coefficient is approximately 1, as we would expect if equation 5 is the correctly specified equation for student test score outcomes. Appendix Table A6 also indicates that the type-specific value-addeds are similarly predictive of test score gains for both rich and poor types and that, while own type-specific value-added is a strong predictor of a child's test scores gains, the type-specific value-added for the other type has no additional predictive power.

[27] These consist of controls for the report card intervention, whose effects are allowed to be different for each type in each year.

[28] To avoid any spurious correlation between the student's own test scores and the school's estimated value-added, I recalculate each school's type-specific value-added excluding the student whose test scores are being predicted.

With these value-added estimates, I can also directly compare value-addeds for rich and poor types in the same schools to see if match-specific quality appears to affect students' outcomes. Figure 2 plots each private school's value-added estimate for rich types against its value-added estimate for poor types. There is a strong correlation between the two (0.61), but the fact that this value is not equal to 1 is not merely due to measurement error. An F-test of the estimated interactions between school fixed effects and an indicator variable for being a rich type in equation 5 rejects the possibility that these effects are jointly equal to 0 with a F-statistic of 1,310 when the outcome variable is mean test scores. While this suggests that match-specific quality plays a role in students' outcomes, it is important to keep in mind that the correlation between value-addeds for rich and poor students is *not* a sufficient statistic for the importance of match-specific quality since it is also affected by schools' equilibrium choices of h_{jt}. In an extreme example, if every school selected a h_{jt} of one-half, the correlation between rich and poor students' value-added would be 1 regardless of the size of β.

5.3 Determinants of School Demand

In this section, I estimate the parameters of equation 4 using a discrete choice model with unobserved school quality in the spirit of Berry et al. (2004). Since my data has repeated observations of schools' characteristics and students' enrollment decisions over time, I now add the t subscript, for the year the data was collected, to equation 4.

Then, the new equation for the utility of a student i of type z in school j and year t is

$$u_{ijzt} = \delta_z VA_{jzn} + \mathbf{\Gamma_z^{indiv}} X_{ijt}^{indiv} + \zeta_{jn} + \epsilon_{ijt}, \tag{6}$$

where X_{ij}^{indiv} is the set of characteristics affecting school choice that vary at the individual-level, consisting of the interaction of school fees and an indicator variable for being a rich type, the effect of distance on rich and poor types, a control for a child having been in school j in the previous period interacted with type, and controls for a boy attending a school marked as an all boys school or a girl attending a school marked as an all girls school.[29] ζ_{jn} is the school fixed effect, which is allowed to vary non-parametrically with the number of private schools in the market, n, and is equal to

$$\zeta_{jn} = \xi_{jn} + \mathbf{\Gamma_z^{school}} X_{jn}^{school}. \tag{7}$$

[29] $\mathbf{\Gamma_z^{indiv}}$ does not include the interaction between school fees and being a low type since including both this control and the interaction of school fees with being a high type would be collinear with the school fixed effect. Instead, I estimate the baseline effect of school fees on school choice, $\mathbf{\Gamma_z^{school}}$, separately.

Here, ξ_{jn} is the school's unobserved quality and X_{jn}^{school} is a school j's average, inflation-adjusted fees under competitive regime n. Therefore, my parameters of interest are the coefficients in a student's utility function: $\{\mathbf{\Gamma_z^{school}}, \mathbf{\Gamma_z^{indiv}}, \delta_{low}, \delta_{high}\}$.

The distinction between equation 6 and equation 7 is important. The variables $VA_{j,poor,n}$, $VA_{j,rich,n}$ and X_{ijt}^{indiv} vary at the individual-level while X_{jn}^{school} varies at the school-level. Therefore, the coefficients $\{\mathbf{\Gamma_z^{indiv}}, \delta_{poor}, \delta_{rich}\}$ can be estimated jointly with the school fixed effects, ζ_{jn}, using a maximum likelihood procedure with individual-year level choice data. Intuitively, including the school fixed effects accounts for any unobserved characteristics of the school that may affect school choice and be correlated with a school's type-specific value-added, and which would otherwise bias the estimates of δ_z. For example, if schools with higher value-addeds also have other attractive features like toilets, the estimates of δ_{poor} and δ_{rich} would be positively biased in the absence of school fixed effects.

$\mathbf{\Gamma_z^{school}}$ is not identified in this procedure since X_{jn}^{school} is collinear with the school effects. For this reason, I separate my estimation into two stages. In the first stage, I estimate $\{\mathbf{\Gamma_z^{indiv}}, \delta_{poor}, \delta_{rich}\}$ using maximum likelihood, and in the second stage, I estimate $\mathbf{\Gamma_z^{school}}$ using the general method of the moments. As I will show in Section 5, it is not necessary to estimate $\mathbf{\Gamma_z^{school}}$ to estimate private schools' equilibrium choices of horizontal quality since the baseline effect of school fees is subsumed by the estimates of ζ_{jn}. Nonetheless, estimates of $\mathbf{\Gamma_z^{school}}$ are useful since they allow us to measure the importance of value-added relative to fees for both high and low types and since the baseline effect of prices should be strongly negative, estimating this coefficient provides an additional check of the structural model. The estimation procedure for $\mathbf{\Gamma_z^{school}}$ is documented in Appendix C.

To allow students' choices of schools to depend on distance, I use the household data rather than the tested data to estimate the demand for schooling. Additionally, to ensure that there is no correlation between the estimated value-addeds and the errors in the discrete choice model, I drop children in the household survey data who also appear in the tested sample. In other words, I estimate the school characteristics $VA_{j,poor,n}$, $VA_{j,rich,n}$ using the tested sample, but use the child characteristics and observed choices in the non-overlapping portion of the household sample to estimate the determinants of school demand.

Identification of $\{\mathbf{\Gamma_z^{school}}, \mathbf{\Gamma_z^{indiv}}, \delta_{poor}, \delta_{rich}\}$

Since I have previously assumed that ϵ_{ijt} is a type 1 extreme value error, the probability that a student i of type z attends school j in year t can be written as

$$p_{ijtz} = \frac{e^{\delta_z VA_{j,z,n} + \mathbf{\Gamma_z^{indiv}} X_{ijt}^{indiv} + \zeta_{jn}}}{\sum_k e^{\delta_z VA_{k,z,n} + \mathbf{\Gamma_z^{indiv}} X_{ikt}^{indiv} + \zeta_{kn}}}.$$

However, recall that for each child in the household survey, I have estimated the probability of being a rich type rather than assigning each child a binary type. Therefore, I write the expression for the probability that a child i attends a school j in year t as:

$$\begin{aligned} p_{ijt} = P(type_i = rich) & \frac{e^{\delta_{rich} VA_{j,rich,n} + \mathbf{\Gamma^{indiv}_{rich}} X^{indiv}_{ijt} + \zeta_{jn}}}{\sum_k e^{\delta_{rich} VA_{k,rich,n} + \mathbf{\Gamma^{indiv}_{rich}} X^{indiv}_{ikt} + \zeta_{kn}}} \\ + (1 - P(type_i = rich)) & \frac{e^{\delta_{poor} VA_{j,poor,n} + \mathbf{\Gamma^{indiv}_{poor}} X^{indiv}_{ijt} + \zeta_{jn}}}{\sum_k e^{\delta_{poor} VA_{k,poor,n} + \mathbf{\Gamma^{indiv}_{poor}} X^{indiv}_{ikt} + \zeta_{kn}}}, \end{aligned} \tag{8}$$

where $P(type_i = rich)$ is the probability that i is a high type that was previously estimated with the logistic lasso regression and $VA_{j,z,n}$ is type z's value-added in school j during competitive regime n.

Using equation (8), I choose the parameters $\{\mathbf{\Gamma^{indiv}_z}, \delta_{poor}, \delta_{rich}, \zeta_{zn}\}$ that maximize the log likelihood function

$$\sum_{ijt} \mathbb{1}_{ijt} log(p_{ijt}),$$

where $\mathbb{1}_{ijt}$ is an indicator variable equal to 1 if i attends school j in year t. Intuitively, this estimation procedure chooses the parameters which make students' observed enrollment decisions most likely. More details of the estimation procedure are described in Appendix C.

Estimates

Table 4 reports the estimates for the key parameters of the utility function. Reassuringly, the directions and relative magnitudes of the coefficients match standard economic intuitions. Both types respond negatively to distance, but poor types are much more sensitive to distance. Both types also respond negatively to fees (measured in 1000s of Rupees), but the wealthy types are somewhat less sensitive to school fees.

The estimates confirm that poor types are much less responsive to match-specific quality relative to rich types ($\delta_{rich} > \delta_{poor}$), suggesting that schools have an incentive to choose instructional levels that benefit rich types. While poor types do respond positively to their predicted test score gains in a school, rich types are much more responsive. An increase in predicted test score gains of 1 standard deviation increases the utility of attending a school for rich types by five times as much as it increases the utility for poor types.

Since coefficients in the utility function are difficult to interpret, I also compare the effects of type-specific value-added for rich and poor types in two other ways. First, I can compare what change in fees is equivalent to a 1 student-level standard deviation increase in value-added. For rich types, an increase in value-added of 1 standard deviation is equivalent to a reduction in fees of 574 Rupees. For poor types, it is only 93 Ruppees. Second, I can calculate the average derivative of each school's enrollment with respect to type-specific value-added and compare these derivatives for the value-addeds of rich and poor types. I find that, on average, a school will increase its student population by six times as much by increasing value-added for rich types relative to poor types.

5.4 Equilibrium Choice of Horizontal Quality

To estimate h_{jt} and β, I assume that schools are observed choosing their equilibrium characteristics. The estimation strategy then consists of two stages. In the first stage, the condition $\frac{\partial s_j}{\partial h^*_{jt}} = 0$ is used to identify h_{jt} for every private school-year observation jt. In the second stage, the estimates of h_{jt} are used to identify β.

In the first stage, I begin by noting that, in equilibrium, for each jt,

$$\frac{\partial s_{jt}}{\partial h_{jt}} = \sum_{it} P(type_i = rich)\frac{\partial p_{ij,rich,t}}{\partial h_{jt}} + (1 - P(type_i = rich))\frac{\partial p_{ij,poor,t}}{\partial h_{jt}} = 0,$$

where

$$\begin{aligned} \frac{\partial p_{ij,poor,t}}{\partial h_{jt}} &= 2\delta_{poor}\beta h_j(p^2_{ij,poor,t} - p_{ij,low,t}) \\ \frac{\partial p_{ij,rich,t}}{\partial h_{jt}} &= \delta_{rich}p_{ij,rich,t}(2\beta - 2\beta h_j)(1 - p_{ij,rich,t}). \end{aligned} \tag{9}$$

Dividing through by 2β produces the expression used to identify h_{jt}:

$$\begin{aligned} &\sum_{it} P(type_i = rich)\delta_{rich}(1 - h_{jt})p_{ij,rich,t}(1 - p_{ij,rich,t}) \\ &+ (1 - P(type_i = rich))\delta_{poor}h_{jt}(p^2_{ij,poor,t} - p_{ij,poor,t}) = 0 \quad \text{for each } jt. \end{aligned} \tag{10}$$

Importantly, equation 10 is now independent of β, and all the terms besides h_{jt} are observed. The probability that a student is a rich type, $P(type_i = rich)$, is given by the logistic lasso regression, and $p_{ij,rich,t}$ and $p_{ij,poor,t}$ can be estimated using the demand system in the previous section. Then, I estimate h_{jt} for all jt by solving for the h_{jt} that sets this expression equal to 0.

Using these estimates of h_{jt}, it is straightforward to estimate β. Manipulating equations

(2) and (3) results in the expression

$$\widehat{VA_{j,rich,n}} - \widehat{VA_{j,poor,n}} = \beta(2h_{jt} - 1), \tag{11}$$

and using this expression, β is identified by a regression of the difference between the observed value-addeds for high and low types on $(2h_{jt} - 1)$. Equation 11 is intuitive: it uses the correlation between schools' equilibrium choices of horizontal quality and the difference in test score gains in a school for rich and poor students to identify β. More details of the estimation procedure are discussed in Appendix D.

Estimates

I estimate that β is 0.328.[30] This implies that horizontal quality can play a large role in students' outcomes; choosing an instructional level that is optimal for rich types will reduce poor types' test scores by 0.328 standard deviations, equivalent to the test scores gains from three-quarters of a year of schooling, relative to choosing the instructional level that is optimal for poor types.

Figure 3 plots the distribution of the estimates of schools' equilibrium horizontal qualities. The average value of h^*_{jt} is 0.80, implying that private schools typically choose instructional levels that strongly advantage rich types over poor types. Figure 3 also reveals that schools' choices of horizontal quality are strongly skewed toward high types with almost no schools selecting a horizontal quality below 0.5. Given the estimate of β, moving a poor student from a private school with the average horizontal quality to one at her optimal instructional level would improve her test scores by 0.210 standard deviations.

The estimates of h^*_{jt} are also consistent with the event study results from Section 3, which show that increasing competition increases inequality in test scores in the private sector. A regression of the estimated h^*_{jt} on the number of private schools in the market shows that an additional private school in the market place increases h^*_{jt} by 0.004 (with a standard error of 0.002). An increase in h would increase test score inequality between rich and poor students, increasing inequality in the private sector.

Figure 4 plots a binscatter graph of the effect of the number of private schools on the equilibrium choice of h^*_{jt} and the implied effects on the mean value-addeds for rich and poor types, with each dot representing the average of the outcome variable for a village with x private schools. These graphs illustrate an interesting testable prediction of the estimates. Increasing the number of private schools in a village increases the test scores for

[30] The standard error from the OLS regression is 0.01, but this does not account for error coming from estimation error in the h_{jt}.

rich types much less than it decreases the test scores of poor types. This is because the average value of h^*_{jt} is already high prior to an entry event, and poor types, whose optimal instructional level is farther from their school's instructional level, suffer greater losses when h_{jt} increases. In contrast, rich types are already close to their optimal instructional level and benefit relatively little from the change in h_{jt}. This prediction also allows us to rule out an alternative explanation for the increase in inequality in test scores that we observe in Section 3. If schools increase quality for everyone but rich types disproportionately benefit from this increase (for example, because they learn faster), this would also drive an increase in inequality in test scores. However, this would be inconsistent with poor types' test scores falling, as well as the lack of a detectable positive effect of competition on the average student's test scores.

Finally, to understand what drives schools to choose instructional levels that favor rich students, I re-estimate schools' equilibrium values of h^*_{jt} after setting $\delta_{low} = \delta_{high}$. If poor students were equally responsive to school quality, the average value of h_{jt} would be 0.46 instead of 0.80. Thus, the fact that schools choose horizontal qualities that so strongly advantage rich students appears to be driven by the fact that rich students are more responsive to quality.

6 Reduced-Form Test of the Structural Model's Predictions

In this section, I report the results of a model independent test of the structural model's prediction that inequality should increase between rich and poor types and that this should be driven by a fall in poor types' test scores rather than an increase in rich types' test scores. In the first subsection, I again exploit the exit and entry of private schools into the education market over time to test how increased school competition affects the different types' test scores in the private sector. In the second subsection, I provide evidence that the estimates from the exit-entry regressions are not driven by selection or omitted variable bias.

6.1 Evidence From School Entry and Exit

To test whether competition increases inequality between rich and poor types and whether this is driven by a fall in the test scores of poor types, I exploit variation in the number of private schools due to private school entry and exit. Formally, to estimate the heterogeneous effects of the number of private schools on the learning of rich and poor types, I estimate the regression

$$
\begin{aligned}
y_{it} = & \rho_0 + \rho_1 num\_pri_{vt} + \rho_2 num\_pri_{vt} \times \mathbb{1}_{rich} + \eta_{zj} + \alpha_{zt} + \omega_{gz} \\
& + \lambda_{gz} y_{i,t-1} + \phi_{gz} y_{i,t-1}^2 + \mathbf{\Gamma X_{vzt}} + \epsilon_{it},
\end{aligned} \tag{12}
$$

where i indexes students, g indexes grades, z indexes types, v indexes villages, j indexes schools, and t indexes years. y_{it} is then a student-year level test score in math, Urdu, or English, α_{zt} are year-by-type fixed effects, ω_{gz} are grade-by-type fixed effects, and η_{zj} are school-by-type fixed effects. λ_{gz} and ϕ_{gz} are grade-by-type specific coefficients on test scores and lagged test scores, and $\mathbf{X_{vzt}}$ includes additional controls at the village by type by year level.[31] Standard errors are clustered at the village-level. In some specifications, I also include controls for student-teacher ratios and peer controls, all of which are allowed to have different effects for rich and poor types. I include these controls since exit and entry may affect both the number of students in a school and the composition of the schools. Peer effects controls consist of a control for the mean and variance of the lagged test scores of the students in a school and a control for the percent of rich students in the school.

This regression allows me to estimate the change in the test scores of poor and rich individuals induced by the entry of a new private school. The coefficients of interest are ρ_1 and ρ_2. Intuitively, controlling for the school-by-type fixed effects and the rich function of lagged test scores means that ρ_1 and ρ_2 are identified by changes to the value-added for a type within a school when another school exits or enters the market.

Table 5 reports the results from this specification for math, Urdu, English, and mean test scores. The odd columns report the results for the basic specification, while the even columns add the additional peer effects and student-teacher ratio controls. Adding an additional private school has a negative effect on average test scores for poor types and a positive effect for rich types (columns 7 and 8), and including the additional controls only strengthens this finding. An additional private school in the market place increases inequality in yearly test score gains by 0.08 standard deviations, and this is mainly driven by a fall in poor types' scores.

The heterogeneous effects documented here are consistent with the results from the structural model. To tease out the proposed mechanism in the structural model – that schools change their instructional level to advantage rich students who desire more advanced instruction at the expense of poor students, I now analyze data on how students perform on specific questions. For the set of questions asked in all four years, I code questions that less than

[31] Just as in the value-added estimation, these controls consist of controls for the report card intervention, whose effects are allowed to vary at the year by type level.

one-third of students answered correctly in the first year as hard and questions that more than two-thirds answered correctly as easy. Now, I re-estimate equation 12 with the share of easy questions answered correctly and the share of hard questions answered correctly as the outcome variables. Since I am combining questions across subjects, I control for lagged mean test scores.

If schools respond to competition by moving their instructional level closer to the needs of more advanced, wealthy students, we expect rich types to perform better on hard questions when there are more private schools in the market and poor types to perform worse on easy questions. Table 6 confirms that this is the case. An additional private school in the market place reduces the share of easy questions poor types answer correctly by 1 percentage point (significant at the 5 percent level) and increases the share that rich types answer correctly by 3 percentage points (significant at the 1 percent level).

6.2 Robustness of School Exit and Entry Results

In this section, I discuss several possible sources of bias in the estimation of ρ_1 and ρ_2 and test whether my estimates are robust to them. I show that (1) the results are robust to an alternative method for selecting the rich and poor types, (2) that the results are not driven by pre-trends and (3) that they are not driven by new students entering the private sector following an entry event.

One concern about the results is that the types were selected using private school students' test scores so that the same students' test scores "appear" on both the left and the right sides of the estimating equation. To show that this does not affect the results, I re-estimate the types using only the test scores of non-private school students to identify the coefficients on the assets in the lasso regression and re-estimate equation 12. Appendix Table A7, which reports these results, shows that they are virtually identical to the results generated when the full sample is used to estimate the types.

An additional concern is that these results, which exploit the exit and entry of private schools, are biased by pre-trends. In Appendix Table A8, I test whether the results are driven by pre-trends in test score outcomes more explicitly. I include the forward lag of the number of private schools and its interaction with being a rich type in equation 12. These forward lags are placebo tests; they test whether the number of private schools in year $t+1$ had an effect on outcomes in year t before the entry or exit event took place. If the main effects are indeed driven by pre-trends, one would expect the estimates of the forward lags to be similar to the estimates of ρ_1 and ρ_2. There is no evidence that this is the case. Across all subjects and for both high and low types, the forward lags are small and statistically insignificant.[32]

Finally, the estimates of ρ_1 and ρ_2 may also be biased if the creation of new private schools leads new students to enroll in private schools who would otherwise attend public schools. These new students may be unobservably worse than the poor students already attending private schools, leading to a negative correlation between the number of private schools in the market and the performance of poor types in private schools. To ensure that my results are robust to this possibility, I re-estimate equation 12, restricting my sample to students who always attend private schools and are observed prior to the exit and entry events. Appendix Table A9 reports the results of this regression. The results are similar: the addition of a private school in the village increases inequality in test scores between low and high types by a statistically significant 0.1 standard deviations.

7 Conclusion

In this paper, I estimate a structural model where schools compete for students by choosing both match-specific and vertical quality. Importantly, the model shows that wealthy students are very responsive to their predicted test score gains when they choose schools, while poor students are much less responsive. Following the intuition of Spence (1975), this causes schools to respond to competition by choosing instructional levels closer to wealthier students' optimal instructional level, to the detriment of poor students.

When a new private school enters the market, within-school inequality in test scores increases in the private sector. This is mainly driven by a decline in poor students' test scores, although wealthy students do benefit. My model-independent estimates imply that an additional private school in the market leads the gap in test score gains between rich and poor students to grow by 0.1 standard deviations. Over the course of five years of primary schooling, with perfect persistence of learning, the magnitude of the gap in levels will be nearly half the size of the black-white test score gap in the United States (Fryer and Levitt, 2004). These results highlight the fact that inequality in test scores between rich and poor students can be driven by inequality within schools, as well as inequality across schools.

The estimation of the structural model delivers two additional, important results. First, private schools choose instructional levels that are highly skewed toward the optimal instructional levels of wealthy students. The average private school in the data chooses a horizontal quality of 0.8, even though half of the private school population desires an optimal instructional level of 0. This suggests that there is substantial misallocation in instructional level

[32] The smaller sample size in Appendix Table A8 relative to Table 5 is due to the fact that the forward lag for the number of private schools is missing in 2007, causing these observations to be dropped.

from the perspective of maximizing students' learning.

Second, the structural estimation delivers an estimate of β, the importance of instructional match in the learning production function. According to this estimate, a student in a school whose instructional level is the opposite of her optimal instructional level will have 0.328 standard deviation lower test scores than a student in a school that perfectly matches her optimal level. This suggests that the misallocation of instructional level could greatly affect students' learning and that interventions that affect the match between schools and students could play an important role in increasing poor students' outcomes. In light of the low levels of learning in many low-income countries, such interventions may be an important policy tool for improving students' outcomes. For instance, interventions that improve the match between instructional level and students such as tracking by ability (Duflo et al., 2011), tutoring, and educational technology (Muralidharan et al., 2016) may yield high returns.

The recent growth of low cost private schools in Pakistan mirrors that of much of the rest of the developing world. Educational markets and enrollment rates in private schooling are similar across South Asia and Sub-Saharan Africa. In these contexts, understanding how market structure affects schools' incentives and students' outcomes is a necessary first step for crafting interventions that improve learning.

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Figures

Figure 1: Event Study Graph of the Effect of Exit and Entry on Variance of Private Sector Test Scores (90% Confidence Intervals)

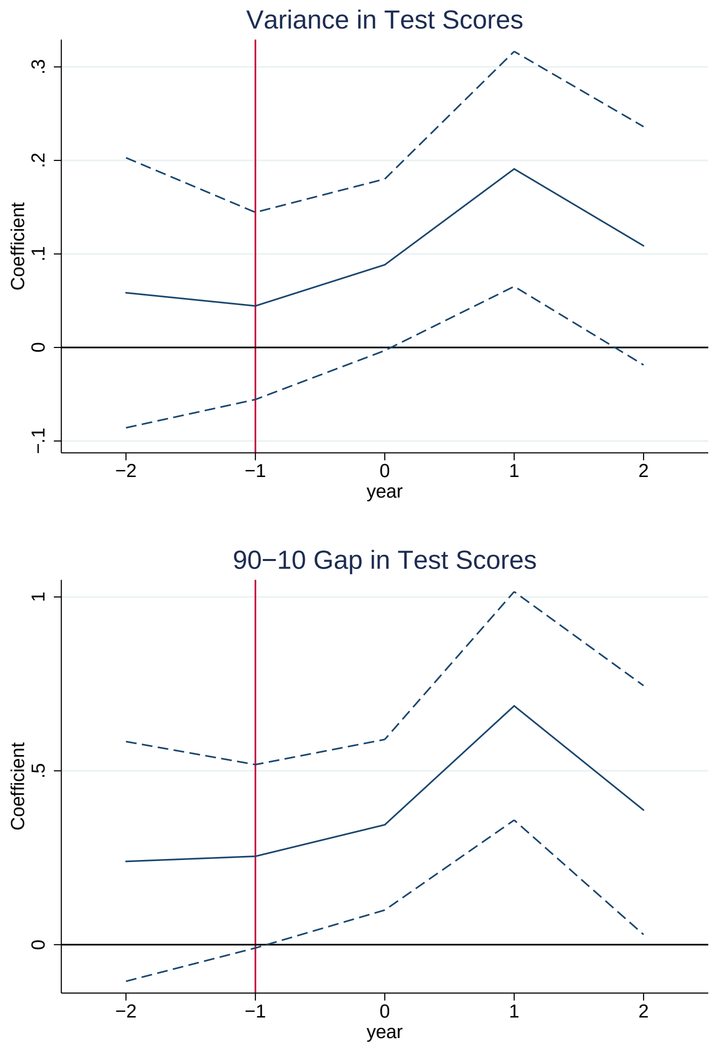


Figure 2: Comparison of School Value-Addeds for Rich and Poor Types

Figure 3: Distribution of Private Schools' Equilibrium Instructional Levels

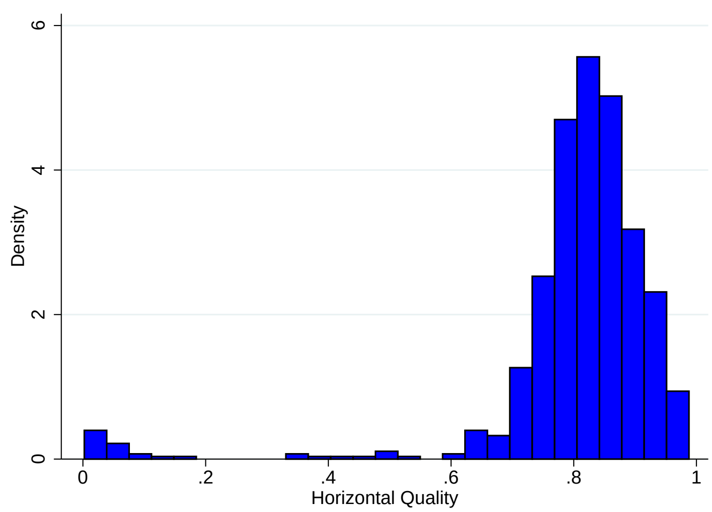
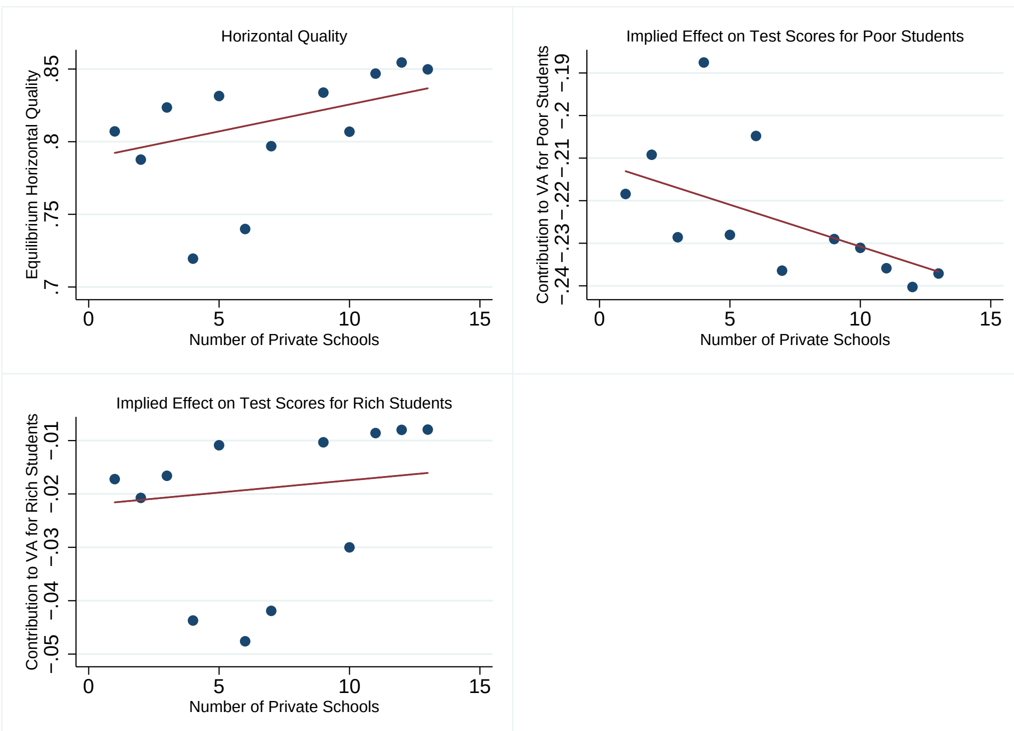


Figure 4: Relationship Between the Number of Private Schools and the Equilibrium Choice of Instructional Level



Tables

Table 1: Effect of Private School Entry on Test Scores in the Public and Private Sectors

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Math		English		Urdu		Mean	
	Private	Public	Private	Public	Private	Public	Private	Public
num_pri_{vt}	-0.088	0.014	-0.079	0.001	-0.056	-0.016	-0.074	-0.0003
	(0.067)	(0.042)	(0.054)	(0.028)	(0.064)	(0.031)	(0.061)	(0.033)
Lagged Test Scores	Y	Y	Y	Y	Y	Y	Y	Y
Grade FE	Y	Y	Y	Y	Y	Y	Y	Y
Village FE	Y	Y	Y	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y	Y	Y	Y
Number of observations	8,732	27,748	8,732	27,748	8,732	27,748	8,732	27,748
Clusters	109	112	109	112	109	112	109	112
Adjusted R^2	0.221	0.174	0.247	0.259	0.237	0.189	0.258	0.231

This table reports estimates of the effect of the number of private schools in the village on test scores for students in the private and public sector separately. The regressions use data from the LEAPS tested children data, and an observation in the regression is at the student-year level. Lagged test score controls consist of the relevant lagged test score and its square interacted with grade fixed effects. Observations are at the student-year level. Standard errors are clustered at the village level. The number of clusters differs by sector because not all villages have a private school in operation in the years that lagged test scores are available.

Table 2: Effect of Private School Entry on the Variance of Test Scores in the Public and Private Sectors

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Variance Math		Variance English		Variance Urdu		Variance Mean	
	Private	Public	Private	Public	Private	Public	Private	Public
num_pri_{vt}	0.084***	0.020	0.095**	0.045	0.098***	0.042	0.077***	0.034
	(0.030)	(0.029)	(0.037)	(0.036)	(0.037)	(0.034)	(0.026)	(0.029)
Village FE	Y	Y	Y	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y	Y	Y	Y
Number of observations	426	447	426	447	426	447	426	447
Clusters	109	112	109	112	109	112	109	112
Adjusted R^2	0.301	0.400	0.252	0.364	0.246	0.313	0.329	0.413

This table reports estimates of the effect of the number of private schools in the village on the variance of test scores in the public and private sectors separately. An observation in the regression is a village-year. The regressions use data from the LEAPS tested children data. Standard errors are clustered at the village level. The number of clusters differs by sector because not all villages have a private school in operation in the years that lagged test scores are available.

Table 3: What Drives the Increase in Test Score Inequality Associated with Entry?

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Variance Math		Variance English		Variance Urdu		Variance Mean	
	No Switchers	School-Level Variance	No Switchers	School-Level Variance	No Switchers	School-Level Variance	No Switchers	School-Level Variance
num_pri_{vt}	0.096**	0.089**	0.075	0.062*	0.123**	0.054	0.085**	0.061**
	(0.044)	(0.039)	(0.047)	(0.034)	(0.054)	(0.047)	(0.036)	(0.030)
School FE	N	Y	N	Y	N	Y	N	Y
Village FE	Y	N	Y	N	Y	N	Y	N
Year FE	Y	Y	Y	Y	Y	Y	Y	Y
Peer Controls	Y	N	Y	N	Y	N	Y	N
Student-Teacher Ratio Control	Y	N	Y	N	Y	N	Y	N
Number of observations	405	801	405	801	405	801	405	801
Clusters	108	108	108	108	108	108	108	108
Adjusted R^2	0.282	0.188	0.217	0.226	0.222	0.269	0.298	0.242

This table examines drivers of the increase in inequality associated with private school entry. Odd columns report estimates of the effect of the number of private schools in the village on the village-level variance of test scores in the private sector, restricting the sample used to calculate the variances to students who never change schools and who were observed in school prior to an entry event. In even columns, the outcome is the school-level variance of test scores in the private sector. So, in odd columns, an observation is at the village-year and in even columns, it is at the private school-year level. Peer controls consist of controls for the variance and average of the lagged test scores of students in a school. The regressions use data from the LEAPS tested children data. Standard errors are clustered at the village level.

Table 4: Determinants of Demand in the Equilibrium Model

	(1) Coefficient	(2) Se
$VA_{j,poor,n} \times \mathbb{1}_{poor}$	0.198	0.284
$VA_{j,rich,n} \times \mathbb{1}_{rich}$	1.093***	0.240
$distance_{ij} \times \mathbb{1}_{poor}$	-1.590***	0.082
$distance_{ij} \times \mathbb{1}_{rich}$	-0.285***	0.053
fee_{jn}	-2.131***	0.524
$fee_{jn} \times \mathbb{1}_{rich}$	0.227*	0.118

This table reports estimates of the determinants of school choice using a discrete choice model where schools are allowed to have time-varying unobserved quality. $VA_{j,poor,n}$ is the average of a school j's value-added for poor types in math, Urdu, and English under competitive regime n, while $VA_{j,rich,n}$ is the average value-added for rich types. Distance is measured in kilometers, and fees are measured in 1000s of Rupees. The coefficients are estimated using the LEAPS household survey data.

Table 5: Effect of Number of Private Schools on Test Scores by Type

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Math		English		Urdu		Mean	
$\mathbb{1}_{rich} \times num\_pri_{vt}$	0.093	0.110**	0.081*	0.078*	0.069	0.060	0.082**	0.088**
	(0.057)	(0.053)	(0.044)	(0.046)	(0.047)	(0.043)	(0.038)	(0.037)
num_pri_{vt}	-0.092**	-0.110***	-0.050	-0.057	-0.026	-0.034	-0.054*	-0.068**
	(0.036)	(0.036)	(0.041)	(0.042)	(0.038)	(0.033)	(0.031)	(0.027)
Peer Controls	N	Y	N	Y	N	Y	N	Y
Student-Teacher Ratio Controls	N	Y	N	Y	N	Y	N	Y
Lagged Test Score Controls	Y	Y	Y	Y	Y	Y	Y	Y
School by Type FE	Y	Y	Y	Y	Y	Y	Y	Y
Grade by Type FE	Y	Y	Y	Y	Y	Y	Y	Y
Year by Type FE	Y	Y	Y	Y	Y	Y	Y	Y
Number of observations	6,788	6,754	6,788	6,754	6,788	6,754	6,788	6,754
Clusters	108	108	108	108	108	108	108	108
Adjusted R^2	0.584	0.583	0.597	0.640	0.629	0.640	0.685	0.703

This table reports estimates of the effect of the number of private schools in the village on test scores for rich and poor types attending private schools. The regressions use data from the LEAPS tested children data. The peer controls consist of the school-level mean and variance of lagged test scores in year t, as well as percent of high types in a school, all of which are allowed to have different effects for rich and poor types. The student-teacher ratio controls consist of a control for the school's student-teacher ratio in year t, which is allowed to have different effects for rich and poor types as well. Lagged test score controls consist of the relevant lagged test score and its square interacted with grade by type fixed effects. Observations are at the student-year level. Standard errors are clustered at the village level.

Table 6: Effect of Number of Private Schools on Performance on Hard and Easy Questions

	(1) % Easy Questions Correct	(2) % Hard Questions Correct
$\mathbb{1}_{rich} \times num\_pri_{vt}$	0.005	0.031***
	(0.010)	(0.011)
num_pri_{vt}	-0.010**	-0.014
	(0.005)	(0.009)
Peer Controls	Y	Y
Student-Teacher Ratio Controls	Y	Y
Lagged Test Score Controls	Y	Y
School by Type FE	Y	Y
Grade by Type FE	Y	Y
Year by Type FE	Y	Y
Mean	0.891	0.334
Number of observations	6,788	6,788
Clusters	106	106
Adjusted R^2	0.314	0.682

This table reports estimates of the effect of the number of private schools in the village on performance on easy and hard questions for rich and poor types attending private schools. The regressions use data from the LEAPS tested children data set. Lagged test score controls consist of the mean lagged test score and its square interacted with grade by type fixed effects. The peer controls consist of the school-level mean and variance of lagged test scores in year t, which are allowed to have different effects for rich and poor types, as well as the percent of high types in a school. The student-teacher ratio controls consist of a control for the school's student-teacher ratio in year t, which is allowed to have different effects for rich and poor types. Observations are at the student-year level. Standard errors are clustered at the village level.

Web Appendix

Appendix A: Descriptive Evidence on the Determinants of School Choice

In this appendix, I provide descriptive evidence that poor students are less responsive to school quality when they make enrollment decisions. I regress measures of information about school quality and self-reported reasons parents choose a school on the probability that a household is rich according to the classification in Section 6. In Appendix Table A10, I estimate five associations using the household survey. First, I regress an indicator variable that is equal to 1 if a child ever changes schools over the course of the survey on the probability that the child is a rich type. Here, I restrict the sample to children who are always enrolled in school between 2004 and 2007. Column 1 shows that there is a strong positive and statistically significant relationship: moving from having a 0 probability of being a rich type to a probability of 1 increases the likelihood that a child changes schools between 2004 and 2007 by 31 percentage points. In column 2, I regress an indicator variable for whether a child's parents' know her teacher's name on the probability that a child is a rich type. Again, there is a strong and statistically significant relationship: moving from a 0 probability of being a rich type to a probability of 1 increases the likelihood that parents know the teacher's name by 26 percentage points. In column 3, I regress an indicator variable equal to 1 if parents report choosing a child's school based on distance on the probability the child is a rich type. Here, the sample size drops substantially since parents were only asked why they chose a given school in 2004. I find that rich types are less likely to choose a school based on distance, though this effect is not significant. In column 4, I run a similar regression with an indicator variable for choosing a school based on quality as the outcome. Column 4 indicates that rich types are significantly more likely to report choosing a school based on quality. Finally, in column 5, I regress a school's estimated type-specific value-added for a household on parents' ranking of that school's quality (from 1-5), the household's probability of being a rich type, and the interaction of these two variables. The interaction term, though marginally significant, indicates that moving from having a 0 probability of being a rich type to a probability of 1 nearly doubles the association between a parent's rankings of school quality and the school's type-specific value-added.

Taken together, the associations reported in Appendix Table A10 show that poor types both report caring less about quality when they make enrollment decisions and have less information with which to make these decisions, consistent with estimates of the determinants of school choice, which show that $\delta_{high} > \delta_{low}$.

Appendix B: Mathematical Appendix

In this appendix, I provide proofs for propositions 2.1 and 2.2. Additionally, I prove proposition A1 which shows that, when δ_{poor} is not sufficiently low, there is no pure strategy equilibrium in the simplified model with two private schools.

Proposition 2.1. *For N=1, there is a unique equilibrium where the single private school chooses* $h^* = \max(1-(-u_o)^{1/2}, 0)$.

Proof. When there is one private school, it maximizes its share when it minimizes the share lost to the public school. The school will discontinuously receive all the rich types as long as $-(1-h)^2 \geq u_o$. It can never receive all the poor types, so it will always choose its location to receive all the rich types at $h \geq 1-(-u_o)^{1/2}$. Then, the school minimizes the loss of the poor types subject to this constraint at $h^* = \max(1-(-u_o)^{1/2}, 0)$. □

Proposition 2.2. *For N=2, if* δ_{poor} *is sufficiently small, the unique equilibrium is* $(h_1, h_2) = (1, 1)$.

Proof. The proof proceeds in two steps. First, I show that $(1,1)$ is an equilibrium for a sufficiently low δ_{poor}. Then I show that no other equilibrium is possible.

Step 1: $(1,1)$ *is an equilibrium.* If school 1 chooses $h_1 = 1$, the only possible best responses for 2 are $h_2 = 0$ or $h_2 = 1$ since, if $h_2 < 1$, 2 will lose all high types and 2 will gain more and more low types the closer it places to 0. Then, it is school 2's best response to locate at 1 if

$$\frac{1}{2} + \frac{e^{-\delta_{poor}}}{e^{-\delta_{poor} u_o} + 2e^{-\delta_{poor}}} > \frac{1}{1 + e^{-\delta_{poor} u_o} + e^{-\delta_{poor}}}.$$

We can see that when δ_{poor} is sufficiently low, this will be the case since the derivative of the left-hand side (LHS) with respect to δ_{poor} is always negative:

$$\begin{aligned} \frac{\partial LHS}{\partial \delta_{poor}} &= \frac{-e^{-\delta_{poor}}(e^{-\delta_{poor} u_o} + 2e^{-\delta_{poor}}) - e^{-\delta_{poor}}(-u_o e^{-\delta_{poor} u_o} - 2e^{-\delta_{poor}})}{(e^{-\delta_{poor} u_o} + 2e^{-\delta_{poor}})^2} \\ &= \frac{(u_o - 1)e^{-\delta_{poor} u_o}}{(e^{-\delta_{poor} u_o} + 2e^{-\delta_{poor}})^2} < 0 \end{aligned} \tag{13}$$

and the derivative of the right-hand side (RHS) with respect to δ_{poor} is always positive:

$$\frac{\partial RHS}{\partial \delta_{poor}} = \frac{-(-u_o e^{-\delta_{poor} u_o} - e^{-\delta_{poor}})}{(e^{-\delta_{poor} u_o} + e^{-\delta_{poor}} + 1)^2} > 0.$$

This shows that there is a single crossing in the profit functions from placing at 1 and 0, implying that there exists a δ^*_{poor} such that, for all $\delta_{poor} < \delta^*_{poor}$, 2's best response is to choose $h_2 = 1$. Since the best response functions of 1 and 2 are symmetric, for δ_{poor} sufficiently small $(1, 1)$ is an equilibrium.

Step 2: $(1, 1)$ is unique. If 1 chooses $h_1 \neq 1$, 2's best response function is $max(h_1 + \epsilon, 1 - (-u_o)^{1/2})$, $\epsilon > 0$, since 2 can take all the rich types and split the poor types by deviating ϵ above h_1 as long as the rich types prefer 2 to their outside option. Since 1 and 2 have symmetric best response functions, it is clear that for both schools to play their best response, at least one school must place at 1. In step 1, we showed that if one school locates at 1, the other school's best response is to also locate at 1 if δ_{poor} is sufficiently low. Therefore $(1, 1)$ is the unique equilibrium. □

Proposition A1. *When $N = 2$ and $\delta^*_{poor} < \delta_{poor} < \delta_{high}$, where δ^*_{poor} is the value of δ_{poor} that equalizes the profits from school 2 placing at 0 or 1 when school 1 places at 1, there is no pure strategy equilibrium.*

Proof. If $\delta_{poor} > \delta^*_{poor}$, this implies that the school 2's best response to school 1's choice of 1 will be to choose 0. However, if school 2 chooses 0, it is no longer school 1's best response to choose 1 since school 1 can choose $h_1 = 1 - (-u_o)^{1/2}$, retaining all the rich types and gaining some of the poor types. However, if 1 chooses any h_1 besides 1, it will no longer be in school 2's best interest to choose $h_2 = 0$. Instead, 2 will choose a location $h_2 = \max(1 - (-u_o)^{1/2}, h_1 + \epsilon)$, $\epsilon > 0$, if $h_1 \neq 1$ and 0 otherwise. Since schools 1's and 2's best response functions are symmetric, we can see that there is no set of locations (h_1, h_2) such that both schools are playing their best responses, and there is no pure strategy equilibrium. □

Appendix C: Estimation of the Determinants of School Choice

In the first subsection of this appendix, I discuss how I estimate the parameters $\{\mathbf{\Gamma_z^{indiv}}, \delta_{poor}, \delta_{rich}, \zeta_k\}$ from equation (4). In the second subsection, I discuss how I estimate the baseline effect of fees, $\mathbf{\Gamma_z^{school}}$, on utility.

Estimation of $\{\mathbf{\Gamma_z^{indiv}}, \delta_{poor}, \delta_{rich}, \zeta_k\}$

I estimate the parameters $\{\mathbf{\Gamma_z^{indiv}}, \delta_{poor}, \delta_{rich}, \zeta_k\}$ that maximize the log likelihood function

$$\mathcal{L} = \sum_{ijt} \mathbb{1}_{ijt} log(p_{ijt}), \tag{14}$$

where $\mathbb{1}_{ijt}$ is an indicator variable equal to 1 if a student i attends a school j in year t and p_{ijt} is the probability i attends j in year t given by equation 8. In practice, I do this using the Artelys Knitro package in matlab to minimize the negative log likelihood function. To reduce computational time, I provide the derivatives of equation 14 with respect to $\theta = \{\mathbf{\Gamma_z^{indiv}}, \delta_{poor}, \delta_{rich}, \zeta_k\}$. For notational simplicity, let X_{ijt} also include $VA_{j,rich,n}$ and $VA_{j,poor,n}$. Then, the derivative of equation 14 with respect to the vector θ is

$$\sum_{ijt} \mathbb{1}_{ijt} \frac{1}{p_{ijt}} \Big(P(type_i = rich) \frac{\partial p_{ij,rich,t}}{\partial \theta} + (1 - P(type_i = rich)) \frac{\partial p_{ij,poor,t}}{\partial \theta} \Big), \tag{15}$$

where p_{ijzt} is the probability that a student i chooses j in year t conditional on that student being type z, and

$$\frac{\partial p_{ijzt}}{\partial \theta} = p_{ijzt} \Big(X_{ijzt} - \frac{\sum_k X_{ikt} e^{\theta X_{ikt}}}{\sum_k e^{\theta X_{ikt}}} \Big)$$

for the elements of θ that are in the utility function for type z and 0 for the remaining elements of θ. To ensure that I find the global maximum of equation 14, I estimate $\{\mathbf{\Gamma_z^{indiv}}, \delta_{poor}, \delta_{rich}, \zeta_k\}$ with 20 randomly chosen start points and choose the parameter estimates that produce the largest value for the log likelihood function.

Finally, I estimate the standard errors using the fact that in general, for maximum likelihood estimation, $\sqrt{C}(\hat{\theta} - \theta^*) \to \mathcal{N}(0, I^{-1})$, where the information matrix $I(\theta)$ is given by the expectation of the outer-products of the first derivatives (given by equation 15) of the log likelihood function and C is the number of observations (here, children). Therefore, the covariance matrix is:

$$\frac{1}{C}\left(\sum_i \frac{\partial\mathcal{L}}{\partial\theta}\frac{\partial\mathcal{L}'}{\partial\theta}\right)^{-1}.$$

Estimation of $\mathbf{\Gamma_z^{school}}$

Having estimated $\{\mathbf{\Gamma_z^{indiv}}, \delta_{poor}, \delta_{rich}, \zeta_{jn}\}$ in Section 5.1, I return to equation 7 to estimate $\mathbf{\Gamma_z^{school}}$, which is the baseline effect of fees on a child's utility in a school. Equation 7 appears to be a linear regression. If school fees are unrelated to a school's unobserved quality, ξ_{jn}, then I can estimate $\mathbf{\Gamma_z^{school}}$ by regressing the estimated school fixed effects ζ_{jn} on X_{jn}^{school}. However, this assumption is unlikely to be satisfied. In theory, profit-maximizing schools with higher ξ_{jn} will charge higher prices. Therefore, to identify $\mathbf{\Gamma_z^{school}}$, I need an instrument for school fees.

Following Nevo (2001) and Hausman (1996), I use a variable that shifts the cost of providing education as my instrument for price. In particular, I use geographic variation in teacher salaries as an instrument for schools' prices. To construct this instrument, I regress teacher salaries in private schools on teacher characteristics as follows:

$$salary_{ijt} = \mathbf{\Upsilon} Z_{it} + \eta_j + \alpha_t + \epsilon_{ijt},$$

where $salary_{ijt}$ is the salary of teacher i in school j in year t, η_j is a school fixed effect, α_t is a year fixed effect, and Z_{it} are teacher characteristics consisting of fixed effects for qualifications, experience, training, and age. I regress salaries on these characteristics to ensure that differences in the cost of teachers are not explained by differences in teacher quality, which could be related to ξ_{jn}. Then, I predict the residual:

$$\widehat{salary}_{ijt} = \widehat{\eta_j} + \widehat{\epsilon_{ijt}}.$$

For each village v, I create the leave-one out average measure $cost_v = \sum_{m\in T, m\neq v} \widehat{salary}_{ijt}$, where T is the set of villages in the same sub-district as v. I use a leave-one out estimator to ensure that differences in teacher salaries are not driven by competition over teachers in village v, which may also be related to ξ_{jn}. The key assumption here is that any one village is too small to change prices in other villages in the sub-district, but villages in the same sub-district market are likely to have the same systematic differences in teacher labor supply. The final instrument is then the interaction of $cost_v$ with an indicator variable for whether a school j is private, controlling for $cost_v$ and whether the school is private. In equation form, the instrument is calculated from the regression

$$cost_v \times I(private)_j = \rho_0 + \rho_1 I(private)_j + \rho_2 cost_v + \mu_j, \tag{16}$$

where $I(private)_j$ is an indicator variable equal to 1 if a school is private and the final instrument is the estimate of the residual, $\hat{\mu}_j$. Therefore, variation in the instrument comes from being a private school in sub-district where private school teachers are more expensive. With this instrument, the parameters of equation 7 can be estimated with the general method of the moments under the assumption that $\mu_j \perp \xi_j$. The moment conditions are given by

$$\Phi = \begin{pmatrix} \xi'_{jn}\hat{\mu}_j \\ \xi'_{jn}\hat{\mu}_j^2 \\ \xi'_{jn}1 \end{pmatrix}$$

where $\xi_{jn} = \zeta_{jn} - \mathbf{\Gamma}_{\mathbf{z}}^{\mathbf{school}} X_{jn}^{school} - c$ and c is a constant. To estimate c and $\mathbf{\Gamma}_{\mathbf{z}}^{\mathbf{school}}$, I again use Knitro. I first solve for the parameters that minimize $\hat{\Phi}'\hat{\Phi}$. Given these parameters, I estimate the optimal weighting matrix, C (for details, see Cragg (1983) and Hansen (1982)). Having estimated C, I re-estimate the parameters, minimizing $\hat{\Phi}'C\hat{\Phi}$. I calculate the standard errors using the standard "sandwich formula" for GMM.

Appendix D: Equilibrium Choice of Horizontal Quality Estimation

In Section 5, I estimate h_{jt} for every private school j in every year t and β using a two stage procedure. In the first stage, to estimate the h_{jt} that satisfy equation 10, I solve

$$\min_{h_{jt}} O_{jt}(h_{jt}) \quad \text{for each } jt,$$

where

$$\begin{aligned} O_{jt}(h_{jt}) = &\Big(\sum_{it} P(type_i = rich)\delta_{rich}(1-h_{jt})p_{ij,rich,t}(1-p_{ij,rich,t}) \\ &+ (1-P(type_i = rich))\delta_{poor}h_{jt}(p^2_{ij,poor,t} - p_{ij,poor,t})\Big)^2. \end{aligned}$$

In practice, this is implemented using the Artelys Knitro package in matlab.

To minimize computational time, I parallelize the loop through jt. Additionally, I provide the solver with the first derivative of the objective function, which is given by

$$\begin{aligned} \frac{\partial O_{jt}(h_{jt})}{\partial h_{jt}} &= 2O_{jt}(h_{jt})\Big(\sum_{it} -P(type_i = rich)\delta_{rich}p_{ij,rich,t}(1-p_{ij,rich,t}) \\ &+ \delta_{rich}(1-h_{jt})(\frac{\partial \rho_{ij,rich,t}}{h_{jt}} - 2p_{ij,rich,t}\frac{\partial \rho_{ij,rich,t}}{h_{jt}}) \\ &+ (1-P(type_i = rich))\delta_{poor}(p^2_{ij,poor,t} - p_{ij,poor,t}) \\ &+ (1-P(type_i = rich))\delta_{poor}h_{jt}(2p_{ij,poor,t}\frac{\partial \rho_{ij,rich,t}}{h_{jt}} - \frac{\partial \rho_{ij,rich,t}}{h_{jt}})\Big), \end{aligned}$$

where $\frac{\partial \rho_{ij,rich,t}}{h_{jt}} = \frac{\partial p_{ij,rich,t}}{h_{jt}}\frac{1}{2\beta}$ and $\frac{\partial \rho_{ij,poor,t}}{h_{jt}} = \frac{\partial p_{ij,poor,t}}{h_{jt}}\frac{1}{2\beta}$, and $\frac{\partial p_{ij,rich,t}}{h_{jt}}$ and $\frac{\partial p_{ij,poor,t}}{h_{jt}}$ are given by equation 9. Additionally, for some schools, the objective function is extremely small, leading to numerical instability. To address this problem, I scale the objective function and its derivative by $\frac{1}{O_{jt}(0)}$.

The estimation procedure for β in the second stage is straightforward. Following equation 11, I simply regress $\widehat{VA_{j,high,n}} - \widehat{VA_{j,low,n}}$ on $(2h_{jt}-1)$, restricting the constant to be zero.

Appendix Tables

Table A1: Summary Statistics for Public and Private Schools

	(1)	(2)	(3)	(4)	(5)	(6)
		Private			Public	
	Mean	SD	N	Mean	SD	N
Fee (Rupees)	1,360	963	1,166	11	155	1,928
Maximum Grade Offered	7.549	1.975	1,166	5.935	1.987	1,925
Student-Teacher Ratio	21.172	13.554	1,168	38.718	33.974	1,924
Has Library	0.392	0.488	1,168	0.224	0.417	1,930
Has Computer	0.266	0.442	1,168	0.010	0.101	1,930
Has Sports	0.349	0.477	1,168	0.110	0.313	1,930
Has Hall	0.195	0.397	1,168	0.069	0.253	1,930
Has Wall	0.962	0.190	1,168	0.658	0.474	1,930
Has Fans	0.942	0.233	1,164	0.476	0.500	1,926
Has Electricity	0.959	0.199	1,167	0.542	0.498	1,930
Number Permanent Classrooms	4.235	4.143	1,166	3.386	3.042	1,928
Number of Semi-Permanent Classrooms	1.854	2.990	1,168	0.664	1.526	1,929
Number of Staff Rooms	0.531	0.532	1,168	0.265	0.476	1,928
Number of Stores	0.428	0.571	1,168	0.269	0.623	1,929
Number of Toilets	0.668	0.851	1,168	0.315	0.744	1,929
Number of Blackboards	7.031	4.457	1,168	5.295	4.151	1,930

This table reports summary statistics for private and public schools in the LEAPS survey from 2004-2007. An observation is a school-year.

Table A2: Summary Statistics for Tested Students in Public and Private Schools

	(1)	(2)	(3)	(4)	(5)	(6)
		Private			Public	
	Mean	SD	N	Mean	SD	N
Female	0.451	0.498	14,202	0.450	0.498	28,499
Age	10.338	1.820	14,200	10.582	1.819	28,496
Grade	4.120	1.027	14,202	4.133	0.988	28,499
Mother Some Primary	0.481	0.500	14,202	0.299	0.458	28,499
Father Some Primary	0.748	0.434	14,202	0.579	0.494	28,499
Beds	0.998	0.044	14,202	0.996	0.060	28,499
Radio	0.632	0.482	14,202	0.541	0.498	28,499
TV	0.761	0.427	14,202	0.589	0.492	28,499
Refrigerator	0.600	0.490	14,202	0.322	0.467	28,499
Bicycle	0.746	0.435	14,202	0.713	0.452	28,499
Plough	0.250	0.433	14,202	0.222	0.416	28,499
Small Ag. Tools	0.697	0.460	14,202	0.720	0.449	28,499
Tables	0.952	0.213	14,202	0.855	0.352	28,499
Chairs	0.952	0.215	14,202	0.846	0.361	28,499
Fans	0.974	0.161	14,202	0.924	0.265	28,499
Tractor	0.159	0.366	14,201	0.115	0.319	28,499
Cattle	0.529	0.499	14,201	0.601	0.490	28,499
Goats	0.527	0.499	14,201	0.663	0.473	28,499
Chicken	0.573	0.495	14,201	0.649	0.477	28,499
Watches	0.972	0.166	14,201	0.958	0.202	28,499
Motor Rickshaw	0.040	0.196	14,201	0.039	0.193	28,499
Motorcycle	0.295	0.456	14,200	0.169	0.375	28,499
Car	0.124	0.329	14,201	0.048	0.215	28,499
Telephone	0.577	0.494	14,201	0.364	0.481	28,499
Tubewell	0.233	0.423	14,201	0.158	0.364	28,499
Math	0.376	0.829	14,202	-0.044	0.972	28,499
Urdu	0.429	0.857	14,202	-0.099	0.972	28,499
English	0.537	0.753	14,202	-0.205	0.939	28,499
Yearly Gains in Math	0.395	0.641	6,828	0.390	0.717	14,402
Yearly Gains in Urdu	0.432	0.581	6,828	0.443	0.670	14,402
Yearly Gains in English	0.350	0.580	6,828	0.391	0.694	14,402

This table reports summary statistics at the student-year level for the sample of tested students in private and government schools in the LEAPS survey from 2004-2007.

Table A3: Summary Statistics for Household Sample of Children Aged 5-15

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
		Private			Public			Not Enrolled	
	Mean	SD	N	Mean	SD	N	Mean	SD	N
Female	0.434	0.496	2,892	0.479	0.500	6,851	0.593	0.491	4,169
Age	9.537	2.865	2,892	9.967	2.821	6,851	10.732	3.413	4,169
Distance to Current School (km)	0.484	0.687	2,892	0.724	0.927	6,851	–	–	–
Tables	0.717	0.451	2,892	0.623	0.485	6,851	0.475	0.499	4,169
Chairs	0.914	0.280	2,892	0.791	0.407	6,848	0.584	0.493	4,168
Fans	0.733	0.443	2,892	0.689	0.463	6,851	0.610	0.488	4,169
Sewing Machine	0.864	0.343	2,892	0.752	0.432	6,851	0.575	0.494	4,169
Air Cooler	0.153	0.360	2,892	0.074	0.262	6,851	0.042	0.201	4,169
Air Conditioner	0.268	0.443	2,892	0.247	0.431	6,851	0.220	0.414	4,169
Refrigerator	0.368	0.482	2,892	0.182	0.386	6,851	0.107	0.309	4,169
Radio	0.554	0.497	2,892	0.476	0.499	6,851	0.390	0.488	4,169
TV	0.524	0.500	2,892	0.419	0.493	6,851	0.318	0.466	4,169
VCR	0.112	0.315	2,892	0.051	0.220	6,851	0.036	0.186	4,169
Watches	0.908	0.290	2,892	0.907	0.290	6,851	0.849	0.358	4,169
Guns	0.129	0.336	2,892	0.071	0.256	6,851	0.052	0.222	4,169
Plough	0.173	0.378	2,892	0.134	0.340	6,851	0.105	0.306	4,169
Thresher	0.175	0.380	2,892	0.091	0.288	6,851	0.055	0.227	4,169
Tractor	0.092	0.289	2,892	0.059	0.236	6,851	0.042	0.201	4,169
Tubewell	0.221	0.415	2,892	0.166	0.372	6,851	0.133	0.339	4,169
Agricultural Machinery	0.263	0.440	2,892	0.239	0.427	6,851	0.206	0.404	4,169
Agricultural Hand Tools	0.660	0.474	2,892	0.657	0.475	6,851	0.618	0.486	4,169
Motorcycle	0.283	0.451	2,891	0.231	0.421	6,851	0.212	0.409	4,169
Car	0.084	0.278	2,889	0.032	0.176	6,851	0.030	0.169	4,169
Bicycle	0.615	0.487	2,891	0.635	0.482	6,851	0.590	0.492	4,169
Cows	0.654	0.476	2,891	0.715	0.451	6,851	0.710	0.454	4,169
Goats	0.485	0.500	2,891	0.570	0.495	6,851	0.602	0.490	4,169
Chickens	0.388	0.487	2,891	0.424	0.494	6,851	0.395	0.489	4,169

This table reports summary statistics at the child-year level for children aged 5-15 in the 1,740 surveyed households in the LEAPS survey from 2004-2007.

Table A4: Association Between Number of Private Schools and Village Characteristics

	(1) 1981 Population	(2) 1998 Population	(3) % Pop. Change	(4) Mean Assets	(5) Percent Own Land	(6) Gini Coefficient
num_pri_{vt}	248.879**	373.904*	0.009	0.017	-0.001	0.001
	(108.864)	(210.127)	(0.025)	(0.036)	(0.010)	(0.008)
Year FE	Y	Y	Y	Y	Y	Y
Village FE	N	N	N	Y	Y	Y
Number of observations	448	448	436	448	336	448
Clusters	112	112	109	112	112	112
Adjusted R^2	0.146	0.119	-0.005	0.964	0.841	0.265

This table reports the association between the number of private schools in a village and village-level measures of population in 1981, population in 1998, change in population between 1981 and 1998, wealth and inequality. In all columns, an observation is a village-year. In columns 1-3, the outcome data comes from the 1981 and 1998 Punjab population censuses. In column 4, mean assets is generated by conducting a principal components analysis of indicator variables for asset ownership and predicting the first component. The outcome, a proxy for wealth, is the village-year mean of this asset measure. In column 5, the outcome is the percent of surveyed households who reported owning land. There are fewer observations for this outcome because the survey did not include information about land ownership in round 2. In column 6, the outcome is the Gini coefficient for the village-year, generated using the wealth measure from the principal components analysis. Standard errors are clustered at the village level.

Table A5: Effect of Private School Entry on Inequality in Test Scores as Measured by the Gap Between the 90th and 10th Percentile

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	90-10 Gap in Math		90-10 Gap in English		90-10 Gap in Urdu		90-10 Gap in Mean	
	Private	Public	Private	Public	Private	Public	Private	Public
num_pri_{vt}	0.168***	0.026	0.230***	0.102	0.098***	0.075	0.216***	0.064
	(0.058)	(0.049)	(0.087)	(0.066)	(0.037)	(0.064)	(0.073)	(0.054)
Village FE	Y	Y	Y	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y	Y	Y	Y
Number of observations	430	447	430	447	426	447	430	447
Clusters	110	112	110	112	110	112	110	112
Adjusted R^2	0.365	0.407	0.307	0.359	0.246	0.256	0.333	0.374

This table reports estimates of the effect of the number of private schools in the village on the gap in test scores between the 90th and 10th percentile student in the public and private sectors separately. An observation is the regression is a village-year. The regressions use data from the LEAPS tested children data. Standard errors are clustered at the village level. The number of clusters differs by sector because not all villages have a private school in operation during the study period.

Table A6: Out of Sample Validation of Type-Specific School Value-Addeds

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Math Score		English Score		Urdu Score		Mean Score	
	Poor Type	Rich Type	Poor Type	Rich Type	Poor Type	Rich Type	Poor Type	Rich Type
$Math\,VA_{poor}$	0.922***	0.101						
	(0.081)	(0.097)						
$Math\,VA_{rich}$	-0.024	0.956***						
	(0.050)	(0.094)						
$English\,VA_{poor}$			0.972***	0.067				
			(0.056)	(0.098)				
$English\,VA_{rich}$			-0.006	0.940***				
			(0.041)	(0.110)				
$Urdu\,VA_{poor}$					0.947***	0.130		
					(0.081)	(0.114)		
$Urdu\,VA_{rich}$					0.041	1.028***		
					(0.059)	(0.107)		
$Mean\,VA_{poor}$							1.012***	0.157
							(0.081)	(0.141)
$Mean\,VA_{rich}$							0.026	1.032***
							(0.056)	(0.137)
Grade × Lagged Test Score Controls	Y	Y	Y	Y	Y	Y	Y	Y
Number of observations	1,247	579	1,247	579	1,247	579	1,247	579
Clusters	100	93	100	93	100	93	100	93
Adjusted R^2	0.595	0.612	0.635	0.630	0.653	0.691	0.698	0.707

This table reports the coefficients for regressions of the test scores of rich and poor students who change schools on their new school's value-addeds for rich and poor types, controlling for the grade-specific effects of lagged test scores and lagged test scores squared. The relevant type-specific value-added for a student was calculated leaving out that students' own outcomes. Standard errors are clustered at the school level.

Table A7: Effect of Number of Private Schools on Test Scores with Alternative Type Assignment

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	Math		English		Urdu		Mean	
$\mathbb{1}_{rich} \times num\_pri_{vt}$	0.089	0.095	0.062	0.070*	0.081*	0.081*	0.079**	0.085**
	(0.058)	(0.061)	(0.039)	(0.040)	(0.048)	(0.048)	(0.040)	(0.042)
num_pri_{vt}	-0.092**	-0.104***	-0.051	-0.057	-0.032	-0.040	-0.056*	-0.067**
	(0.037)	(0.035)	(0.048)	(0.047)	(0.036)	(0.032)	(0.034)	(0.028)
Peer Controls	N	Y	N	Y	N	Y	N	Y
Student-Teacher Ratio Controls	N	Y	N	Y	N	Y	N	Y
Lagged Test Score Controls	Y	Y	Y	Y	Y	Y	Y	Y
School by Type FE	Y	Y	Y	Y	Y	Y	Y	Y
Grade by Type FE	Y	Y	Y	Y	Y	Y	Y	Y
Year by Type FE	Y	Y	Y	Y	Y	Y	Y	Y
Number of observations	6,788	6,788	6,788	6,788	6,788	6,788	6,788	6,788
Clusters	106	106	106	106	106	106	106	106
Adjusted R^2	0.572	0.585	0.597	0.602	0.631	0.643	0.686	0.704

This table reports estimates of the effect of the number of private schools in the village on test scores for rich and poor types attending private schools. Here, types are generated by running the lasso regression of test scores on assets for only non-private school students rather than all tested students. The regressions use data from the LEAPS tested children data. The peer controls consist of the school-level mean and variance of lagged test scores in year t, as well as the percent of rich types in a school, all of which are allowed to have different effects for rich and poor types. The student-teacher ratio controls consist of a control for the school's student-teacher ratio in year t, which is allowed to have different effects for rich and poor types. Lagged test score controls consist of the relevant lagged test score and its square interacted with grade by type fixed effects. Observations are at the student-year level. Standard errors are clustered at the village level.

Table A8: Tests for Pre-Trends in the Effect of Number of Private Schools on Student Outcomes

	(1) Math	(2) English	(3) Urdu	(4) Mean
$\mathbb{1}_{rich} \times num\_pri_{v,t+1}$	-0.018	-0.002	-0.004	-0.006
	(0.018)	(0.009)	(0.008)	(0.009)
$num\_pri_{v,t+1}$	0.008	-0.018	-0.00007	-0.008
	(0.015)	(0.021)	(0.015)	(0.010)
Peer Controls	Y	Y	Y	Y
Student-Teacher Ratio Controls	Y	Y	Y	Y
Number of Private Schools Controls	Y	Y	Y	Y
Lagged Test Score Controls	Y	Y	Y	Y
School by Type FE	Y	Y	Y	Y
Grade by Type FE	Y	Y	Y	Y
Year by Type FE	Y	Y	Y	Y
Number of observations	2,622	2,622	2,622	2,622
Clusters	104	104	104	104
Adjusted R^2	0.567	0.625	0.627	0.712

This table reports estimates of the effect of the forward lagged number of private schools in the village on test scores. The regressions use data from the LEAPS tested children data. Lagged test score controls consist of the relevant lagged test score and its square interacted with grade by type fixed effects. Number of private schools controls consist of num_pri_{vt} and its interaction with $\mathbb{1}_{rich}$. The peer controls consist of the school-level mean and variance of lagged test scores in year t, as well as the percent of high types in a school, all of which are allowed to have different effects for rich and poor types. The student-teacher ratio controls consist of a control for the school's student-teacher ratio in year t, which is allowed to have different effects for rich and poor types. Observations are at the student-year level. Standard errors are clustered at the village level.

Table A9: Effect of the Number of Private Schools on Student Outcomes for Students Who Always Attend Private Schools

	(1) Math	(2) English	(3) Urdu	(4) Mean
$\mathbb{1}_{rich} \times num\_pri_{vt}$	0.112*	0.101**	0.098*	0.112**
	(0.067)	(0.050)	(0.050)	(0.048)
num_pri_{vt}	-0.107**	-0.061	-0.057	-0.078**
	(0.042)	(0.058)	(0.031)	(0.034)
Peer Controls	Y	Y	Y	Y
Number of Private Schools Controls	Y	Y	Y	Y
Lagged Test Score Controls	Y	Y	Y	Y
School by Type FE	Y	Y	Y	Y
Grade by Type FE	Y	Y	Y	Y
Year by Type FE	Y	Y	Y	Y
Number of observations	5,845	5,845	5,845	5,845
Clusters	108	108	108	108
Adjusted R^2	0.597	0.609	0.652	0.715

This table reports estimates of the effect of the number of private schools in the village on test scores for students who always attend private schools during the sample period and who are observed in school prior to the exit or entry event. The regressions use data from the LEAPS tested children data. Lagged test score controls consist of the relevant lagged test score and its square interacted with grade by type fixed effects. The peer controls consist of school-level mean and variance of lagged test scores in year t, as well as the percent of high types in a school, all of which are allowed to have different effects for rich and poor types. Number of private schools controls consist of $num\_pri_{v,t}$ and its interaction with $\mathbb{1}_{rich}$. Observations are at the student-year level. Standard errors are clustered at the village level.

Table A10: Knowledge of Educational Quality and Determinants of School Choice

	(1) Changed Schools	(2) Knows Teacher's Name	(3) Chose School for Distance	(4) Chose School for Quality	(5) Mean School VA
$P(type_i = rich)$	0.310***	0.261***	-0.162	0.574***	0.153*
	(0.059)	(0.048)	(0.104)	(0.090)	(0.081)
$rank_{ij}$					0.050***
					(0.010)
$P(type_i = rich) \times rank_{ij}$					0.043*
					(0.023)
Mean	0.347	0.532	0.427	0.210	0.022
Observation Level	Child	Child-Year	Child	Child	Parent-School-Year
Number of observations	5,621	13,645	2,873	2,873	22,826
Clusters	1,694	1,695	1,153	1,153	684
Adjusted R^2	0.008	0.005	0.002	0.038	0.031

This table reports descriptive statistics on rich and poor types' knowledge of educational markets and the determinants of their enrollment decisions in the household survey data. Column 1 regresses an indicator variable for changing schools at least once over the course of the study period on the probability of being a rich type for children who were always enrolled in school; each observation is a child, and the standard errors are clustered at the household level. Column 2 regresses an indicator variable for whether a parent knows a child's teacher's name on the probability of being a rich type; an observation is a child-year, and the standard errors are clustered at the household level. Column 3 regresses an indicator variable for if a parent reports distance is the main reason they choose their child's school on the probability of being a rich type. An observation is a child, since the question was only asked in round 1, and the standard errors are clustered at the household level. Column 4 regresses an indicator variable for if a parent reports quality is the main reason they chose their child's school on the probability of being a rich type. As before, an observation is a child, and the standard errors are clustered at the household level. Column 5 regresses a school's expected value-added for a household on parents' assessment of the school's quality, the household's probability of being a rich type, and their interaction. Each observation is at the parent-school-year level, and standard errors are clustered at the school level.